Volume VII. *August, 1898.* *Number 2.*

THE

PHYSICAL REVIEW.

THE EFFECT OF THE FIBROUS STRUCTURE OF SHEET IRON ON THE CHANGES IN LENGTH ACCOMPANYING ITS MAGNETIZATION.

By Edward Rhoads.

General Description of the Phenomenon.

THE subject of the change in dimensions which a piece of one of the magnetic metals undergoes when magnetized is of considerable interest as offering a possible help in the effort to get a closer insight into the nature of magnetism. That there is such a change was discovered by Joule in 1847. The subject has since been studied by many observers, notably by Bidwell and Nagaoka, and as a result we have a good knowledge of the manner in which the changes follow the application of magnetizing force, accompanying the magnetization.

The case which has been most studied is the change in length in the direction of uniform magnetization, as in a long thin cylinder magnetized by a uniform field in the direction of its length or a cylindrical shell magnetized in the direction of its circumference. With such a piece it is found that, in iron, the length at first increases as the magnetic force increases, the change being proportional to the square of the magnetization. Somewhat after passing the point of maximum permeability, however, the length reaches a maximum, and as the field is increased still further a shortening of the piece takes place. This shortening is proportional to the increase of field until quite high fields are reached when it becomes

less in proportion, and finally the bar assumes a minimum length which it maintains in all higher fields. The maximum elongation does not exceed five millionths of the whole length nor the contraction twelve millionths. They differ widely according to the quality, hardness, etc., of the specimen, and are especially dependent, as shown below, upon the "grain." But for any one specimen, as long as its other properties are not altered, the changes are always the same under the same conditions.

Nickel differs from iron in that it never shows any elongation but contracts even in the weakest fields. Cobalt shows a contraction in weak fields followed at higher fields by an elongation which still continues in the highest fields that have been applied to it.

If, instead of continuing to increase the field from any point, it is decreased, the lengthening or shortening which the bar was undergoing is in general reversed but shows a hysteresis effect. That is to say, if the bar was lengthening with increasing field, then when the field begins to decrease it will shorten, but this shortening will not be as rapid in proportion to the change of field as the lengthening was. In looking at the curves representing cyclic change of length it is well to bear in mind that the change in length, unlike the magnetization, is independent of the sense of the magnetizing force which causes it. Thus in taking a cyclic magnetization curve when the field has been reduced to zero, and is being increased in the opposite direction, the magnetization is first reduced to zero and then begins to *decrease* below zero. The change of length on the other hand first becomes zero, and then begins to *increase* again. Hence the relation which the two curves bear to each other stands out more clearly if we imagine the lower half of the magnetization curve drawn above the line of zero magnetization instead of below.

CONNECTION BETWEEN THE MAGNETIZATION AND THE CHANGES IN LENGTH PRODUCED BY IT.

The most striking feature of this relation is the fact that the field at which the minimum change of length is reached is the same as that in which the magnetization becomes zero, *i. e.*, the coersive force. Also it appears, as already mentioned, that in low fields the change of length shows a close proportionality to the square of the

magnetization. The proportionality, however, is not exact, as is at once evident from the fact that the bar does not quite regain its original length even when the magnetization has become zero. It is possible that this, together with the closely allied fact shown by Nagaoka that the factor of proportionality is different in the ascending and descending curve, may be due to a hysteresis effect in the action of the apparatus used, or to mechanical hysteresis in the specimen. If this be the case it may be that the change of length is rigidly a function of the magnetization; or in any case of both the magnetization and the field, of which in weak fields the term containing the magnetization squared becomes all important. But until this is shown, the fact remains that, all other things being equal, the length may still be different under the same magnetization arrived at differently. So all that can be said is that the magnetization and change of length both follow from the same causes, but that apparently neither is rigidly connected with or due to the other.

In the language of the molecular theory this might be expressed by saying that changes in length result from changes in the grouping of the molecular magnets, which changes may or may not result in producing in each group an excess pointing in any direction, *i. e.*, producing a state of magnetization. The extent to which these changes in the grouping may take place without affecting the magnetization is well shown in a recent paper by Fromme.[1]

It is not the object of this paper to discuss the connection between the magnetic change in length and the magnetization; but since both quantities separately have been plotted against the field in the experiments about to be described, it seemed necessary to say this much to show the reason why. And in passing I may perhaps add that I think the very interesting question of this connection might be successfully attacked by taking sets of progressively graded cyclic curves for both phenomena plotted to the magnetizing force and taken between identical limits. Lines could then be ruled on both parallel to the axis of zero field so that, by picking out their points of intersection with the curves, new curves could be drawn

[1] Fromme, Wied. Ann., Vol. 61, p. 55, 1897.

representing the relation between the change of length and magnetization at various constant fields.

PLAN OF THE EXPERIMENT.

The experiments described in this paper have for their object the determination of the effect which the grain, or fibrous structure of sheet iron due to the rolling process by which it is made, exerts on the magnetic changes in length and on the magnetization. Both the subject and the method of studying it were suggested by Professor Rowland. Two strips 3 cm. wide and 91.5 cm. long were cut from a long sheet of transformer iron 0.035 cm. thick. Four were cut across the plate and two along the plate, as shown in the central diagram in Fig. 2. These strips were bent into tubes around a brass bar and measurements were made of the change in a length of 62.9 cm. of each when subjected to gradually increasing and decreasing uniform fields between limits of about 60 C.G.S. Two specimens, one of each kind, were annealed and curves taken as before. Besides this, one specimen of each kind was examined for change of length in increasing fields up to 915 C.G.S. and Young's modulus of elasticity was measured for one of each.

The sheet from which all the specimens were cut was 274 cm. long by 91.5 by .035 cm. thick. It was obtained for me by Dr. Ames through the kindness of Mr. A. L. Rohrer of the General Electric Company, who stated that it represents probably as good a quality of iron for use in armatures as is in the market.

THE APPARATUS AND PRECAUTIONS NECESSARY.

The apparatus for measuring change in length was a modification of that which has been used in this laboratory for several years past and has been several times described.[1] In the figure it is shown in section except the upper part which is in elevation. It consists of a long, hollow solenoid *A*, water jacketed on the inside, which stands vertical. Within this is hung the specimen tube *B*, with a shorter brass tube *C* around its middle which is clamped to it at the lower end by two set-screws *E*. Inside the specimen tube about as far from its upper end as the clamp is from the lower, a

[1] More, loc. cit. Gallaudet, loc. cit. Brackett, loc. cit.

short brass bar *D* is screwed. This bar has a tongue running up inside the tube, and ending in a stirrup formed by the little steel knife edge *F*. The brass tube also has an arm *G* extending out

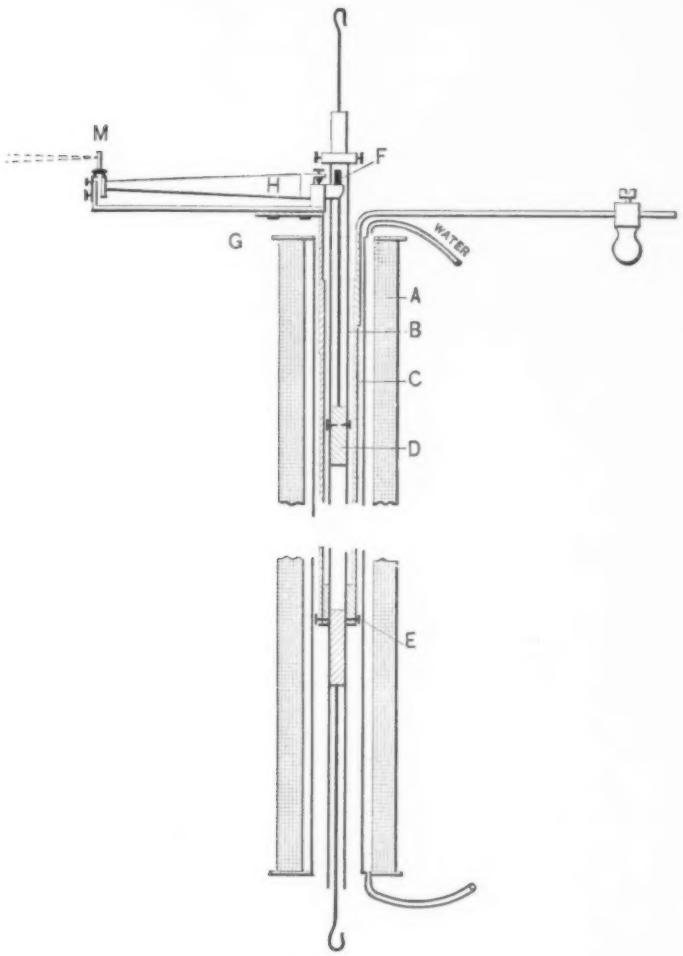

Fig. 1.

the end of the solenoid and a lever rests by means of a knife-edge on a little steel table carried by it, and passes beneath the knife-edge in the stirrup. The other end of the lever extends outwards, and its motion is accurately determined by means of the tilting mirror shown at *M*, and a telescope and scale.

The whole arrangement must hang free of the solenoid by a suspension which transmits no vibrations, for the magnifying power is so great (about 25,000) that the slightest vibration spoils the definition. When hung directly from a weight of about 60 pounds, which in turn was hung by a strong brass coiled spring from a bracket near the ceiling, the scale was usually blurred out except at night. The vibrations seemed to be very small, probably almost sound vibrations, and they still came down even when the spring was suspended by a cotton rope passing over rubber and cotton pads. But when the apparatus was suspended from this system by means of a number of thin rubber bands all vibration effects were done away with, even though the whole building was strongly shaken. I mention this particular because it is customary to make measurements of this kind only at night, a very inconvenient and unnecessary practice.

Another troublesome influence which has always to be considered in such work is the variation of temperature. For rough experiments changes of length arising from this cause may be separated from those due to magnetization by simply taking a reading on the length-scale before, and immediately after, making a known change in the magnetizing current. Since the change in length with the temperature shows itself by a steady and comparatively slow drift of the scale, while the magnetic change is almost instantaneous, this affords a means of separation. For more accurate work there are two methods available. Either the effects of temperature changes may be eliminated by compensation or the temperature may be maintained constant. I found the latter more convenient, though there is also partial compensation or rather over-compensation in the apparatus, for what is really measured is the change in relative length of the brass and iron tubes. This causes an apparent contraction of the iron of about 60×10^{-7} of the whole for an increase of 1° C., so that in order to have no greater errors than $.5 \times 10^{-7}$ the temperature must not vary more than 0°.01 C. and this in spite of the heating of the magnetizing solenoid. The tap water was found to be not at all constant; so it was heated to a temperature about equal to that of the room by an arrangement on the principle of Ostwald's regulator. After passing this it entered

a large barrel supported near the high ceiling, from which it could be drawn off through the water jacket. The water was usually kept running in and out for several hours before a curve was taken. Then the supply was turned off, and the barrel well stirred; after which the water would continue to run out through the jacket for an hour or two without varying more than 0°.01. The regulator got to working so well toward the end of the time that the supply did not need to be turned off at all. The water jacket could not prevent the heat due to high currents from finding its way through in the course of time, but by hurrying over the higher fields in taking the cyclic curves trouble in them was avoided. For the very high currents used in taking the curves given in Fig. 5, the change of length with the time after turning on the current was carefully measured; and it was found to take so long for the heating to begin that there was no trouble in getting at least one reading first. It took only a moment to take a reading; for, the current being continuously increased, there was no vibration of the lever. After each reading a current was passed in the opposite direction, the whole apparatus was allowed to cool down for a while, and the next reading could be taken using the maximum length as a reference point.

Before taking each curve the specimen was demagnetized by an alternating current. The vertical component of the earth's field was roughly compensated for in the later experiments by a single layer of wire outside the magnetizing coil, through which a constant current was kept flowing. When the alternating current was passed, an outstanding field of one tenth of the vertical component was found to reverse the strongest previous magnetization.

If there is a strong initial magnetization in the specimen before a cyclic change of length curve is taken, the cycle will not be symmetrical. A small initial magnetization, however, affects the first ascending branch of the curve only. Such slight lack of symmetry as is shown in the curves here given is, in all probability, due either to this cause or to temperature changes.

Magnetization curves for each specimen were taken under exactly the same conditions as change of length curves, often several of each alternately on the same specimen. The electrical connections were

made as follows: A coil of 1,200 turns of fine wire, wound on a paper tube, was placed over the specimen tube before insertion in the apparatus. This coil was put in circuit with a Rowland D'Arsonval ballistic glavanometer of 1,000 ohms resistance and complete period of 16 seconds, also a standardizing coil wound on a long solenoid, and a reversing switch. For damping, the galvanometer could be disconnected by touching a rocking mercury switch which put in circuit with it a coil in which a magnet slid. The galvanometer was often standardized during the course of the experiments and was found not to alter sensibly. The torsional rigidity of its suspension ribbon must, however, have somewhat decreased during the taking of a curve, for in each case the magnetization apparently reached a slightly higher value the second time the maximum field was applied; but the change was too small to be observed in standardizing. The throws as read were reduced to arc, added up, corrected for the induction through the empty secondary, and lastly reduced to absolute units. The magnetizing solenoid had in its circuit a reversing switch, an iron wire rheostat and a German silver rheostat of about 240 ohms resistance fitted with a mercury connection board so arranged that a copper sulphate resistance could be placed in parallel with each step so as to continuously cut it out or include it.

The current in the solenoid was at first gradually increased from zero by increasing the resistance in a German silver shunt across its terminals, the steps of which were continuously included by the method referred to. When all the shunt resistance had been put in and it had been disconnected, the current was further increased by cutting out the 240 ohms series resistance. This arrangement made it possible to hold the current accurately constant at any value and then rapidly, but continuously change it to another at which it would also remain constant till again changed. It is made necessary by the use of the ballistic galvanometer for taking step by step curves with gradual change of field, and was suggested to me by Dr. Bliss. The circuit could either be connected with the storage batteries, of which twenty-five cells in good condition were used, or with the alternator. For the very high currents used in taking the curves shown in Fig. 5 two direct current generators and a number of cells were placed in series giving 300 volts. The sole-

noid was of No. 18 copper wire wound in seven layers, 84 cm. long and 10.5 ohms resistance and gave 45.7 C. G. S. units of field per ampere. The currents were measured by Weston standard ammeters of appropriate capacity.

From the dimensions of the specimens it will be seen that their length is about 500 times the diameter of a wire of the same cross section, so the demagnetizing force of the ends is inconsiderable. To vertify this fact a coil of ten thousand turns of very fine wire wound on a glass tube was placed inside a specimen tube and connected to the galvanometer. Then a large current was reversed in the magnetizing solenoid. The resulting throw was compared with that given when the specimen tube was removed while everything else remained the same. The ratio of the whole throw to the increase is the ratio of the whole field to the demagnetizing field due to the ends. Both this experiment and a rough calculation pointed to about $\frac{1}{8}$ of a C. G. S. unit of demagnetizing field for an intensity of magnetization of 1000 lines per sq. cm.

Description of Curves Taken.

The first two specimens examined were numbered 1 and 3. Their position in the plate is shown in Fig. 2 in the central diagram. The

Table I.

Changes in length and intensity of magnetization, of Specimen No. **3**, *unannealed; in cyclic field.*

H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$
1.737	.51	46.16	44.59	1.51	12.78	50.27	43.19
2.194	1.53	54.84	43.70	0	6.13	54.84	42.93
3.428	5.11	41.13	46.25	1.05	2.30	36.56	45.74
5.027	10.22	36.56	46.77	1.37	2.04	23.54	46.00
9.140	22.49	25.135	47.02	1.83	2.55	17.59	44.47
16.45	36.80	16.45	44.47	3.20	6.13	9.14	36.03
22.48	41.91	9.14	36.29	4.84	11.24	6.31	29.13
36.42	45.23	5.347	26.58	9.14	23.51	3.84	21.46
41.13	45.23	3.153	18.91	16.91	36.80	0	6.13
				23.35	41.91	1.14	2.04
				36.01	44.47	1.74	1.92
				41.13		1.87	2.04
				45.70	43.96	2.70	3.58
						4.25	8.69

TABLE I.—*Continued.*

H	*I*	*H*	*I*	*H*	*I*
1.92	354	17.59	1338	9.05	740
2.70	519	9.14	1229	17.55	936
3.75	696	4.52	1073	30.66	1039
4.94	822	1.92	919	54.84	1110
9.14	1074	0	702	31.21	1062
17.37	1276	1.19	299	17.55	1004
29.70	1382	1.87	30	9.14	899
54.84	1453	3.11	264	3.15	683
30.80	1401	4.16	422	0	400
				1.37	8.5
				3.02	594
				4.94	863
				10.96	1190

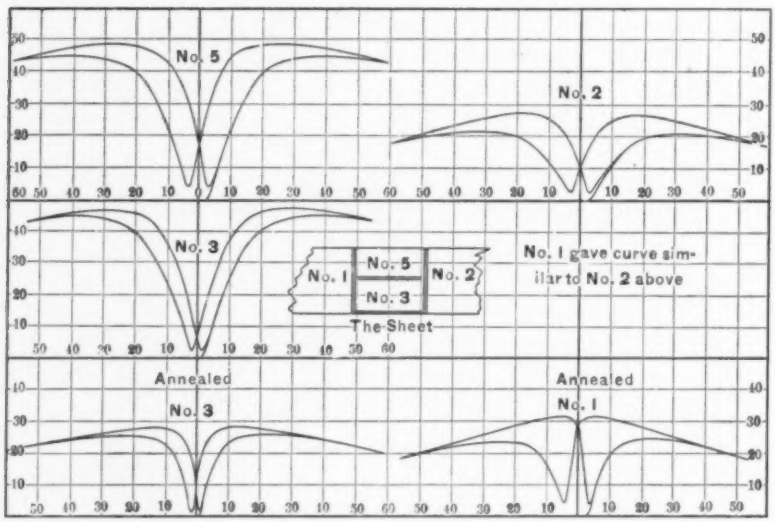

Fig. 2. Cyclic change of length curves for specimens cut out as indicated in the central diagram.

Ordinates = ten millionths of the length.
Abscissas = magnetic field c. g. s.

change of length cycle for No. 3 is plotted on the same sheet, the values being given in Table I. That for No. 1 is not given. As it was exactly like No. 2 taken later except that the coercive force was about 1 C. G. S. unit greater and the retentiveness very slightly

TABLE II.

Values of intensity of magnetization of Specimen No. **1**, ***unannealed in cyclic field.***

H	I	H	I	H	I	H	I
1.65	54	33.36	1355	4.35	104	11.88	970
1.83	66	18.97	1305	6.50	446	5.70	877
3.34	255	12.02	1254	11.88	762	3.20	799
4.52	450	6.50	1178	18.83	914	0	622
9.64	937	3.66	1097	33.36	1036	1.83	390
18.83	1193	0	894	55.75	1116	2.83	110
33.36	1320	1.60	696	33.36	1068	3.79	231
55.75	1401	3.61	93	18.97	1016	6.50	757
						11.88	1096

TABLE III.

Changes in length and intensity of magnetization of Specimen No. **2**, *unannealed.*

H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$
2.33	.05	11.88	25.86	14.72	17.75	7.86	18.66
4.11	1.52	99.55	25.00	35.97	21.29	3.20	10.39
6.19	5.07	4.20	20.28	45.70	20.28	0	3.55
9.51	10.75	1.83	13.31	59.41	17.75	1.83	2.94
16.91	17.75	0	10.04	35.65	23.73	3.24	3.04
22.85	20.28	1.37	5.07	18.65	26.46	4.71	5.07
38.85	20.69	2.74	2.53	12.80	26.36	9.46	12.17
59.41	17.29	4.11	5.07	9.51	25.35	22.39	20.79
24.22	25.60	7.86	10.14		23.42		
18.28	26.36	9.51	12.47				

TABLE III.—*Continued.*

H	I	H	I	H	I	H	I
1.74	67.2	9.50	1064.0	9.37	845	0	710
3.20	276.8	3.20	922.6	18.28	1040	.82	592
4.25	470.4	1.83	859	52.78	1203	1.78	374
6.63	696.0	0	714	59.41	1211	2.33	106
9.37	841.6	.87	597	52.56	1202	3.20	128
18.28	1040.0	1.78	373	18.28	1118	4.39	422
53.01	1203.2	2.33	108	9.50	1058	6.63	698
59.41	1219.2	3.15	125	3.20	918	9.37	854
52.60	1206.4	4.34	414				
18.28	1124.8	6.63	690				

greater, it has not been thought worth while to reduce to absolute units and plot. The magnetization curves for these two specimens are given in Fig. 3 and in Tables I. and III. The initial magnetization shown is due to the fact that the earth's field was not compensated for during the previous demagnetization. The initial length, *i. e.*, zero of change of length, has been corrected in the case of No. 3 for this.

The very marked difference between the cycles for these two specimens is apparent. They differ in the maximum length reached, in the field in which it is reached, in retentiveness, in coercive force, and the differences are similar in the magnetization curves. But it

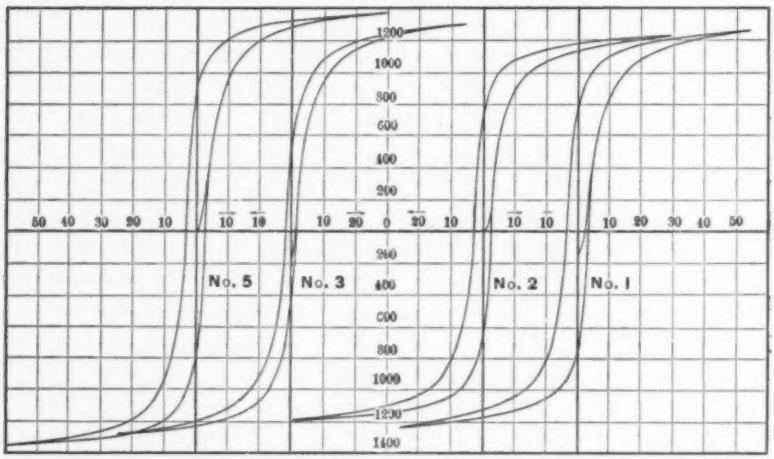

Fig. 3. Cyclic magnetization curves for specimens cut out as indicated in diagram in Fig. 2.

Ordinates = lines of magnetization per sq. cm.
Abscissas = magnetic field c. g. s.

might well be objected that merely chance differences in the specimens would account for this, especially as one comes wholly from the edge of the plate while the part of the other which is examined comes from the middle. Therefore specimens 2 and 5 were cut out and examined; the results are plotted in the same figure (2), and given in Tables III. and IV. Number 2 showed, as stated, almost identical changes in length with No. 1; so it seems fair to assume that any other specimen cut parallel to them from the space between

TABLE IV.

Change in length and intensity of magnetization of Specimen No. **5**, *unannealed.*

H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$
2.79	12.68	59.41	47.66	6.86	32.96	9.51	27.88
3.66	7.61	23.86	48.11	9.37	38.03	6.40	17.75
4.57	3.40	16.00	47.66	14.40	41.47	3.66	17.34
7.77	5.58	9.55	43.50	18.28	43.60	0	7.61
9.19	12.68	4.57	39.04	23.81	44.72	1.78	4.06
13.71	21.24	2.29	32.14	37.93	44.62	2.47	3.09
18.26	32.85	0	.51	59.41	43.35	4.02	4.06
22.85	38.03	.37	1.57	32.45	48.67	5.94	9.63
28.33	41.83	1.78	4.06	23.99	47.66	7.50	15.21
36.56	44.51	3.20	15.62	18.28	44.51	9.60	21.80
45.70	42.99	4.57	20.28		35.24		

TABLE IV.—*Continued.*

H	I	H	I	H	I	H	I
1.78	70.3	31.53	1303.5	4.34	411	9.46	1212
3.15	264.6	14.17	1232.6	6.63	744	3.20	1059
4.25	467.6	9.37	1175.1	9.28	938	0	855
6.58	728.0	3.20	1032.1	18.19	1184	.87	751
9.23	921.3	0	825.2	31.62	1294	1.78	556
23.76	1222.3	.87	720.2	52.10	1359	2.33	256
52.33	1339.2	1.78	530.5	59.41	1377	3.15	39
59.41	1335.4	2.33	232.5	52.05	1365	4.34	363
52.10	1343.6	3.15	58.4	23.76	1302	9.37	926

would show the same. Number 5, cut from the middle of this space in the other direction, showed a maximum length equal to that of No. 3, and occurring in the same field. It seems, therefore, that the difference in the maximum change of length and the field in which it is reached must be due to differences in the direction of the grain. The differences in hysteresis and coercive force, however, seem to be merely dependent upon the position of the specimen in the plate. The coercive force of specimen 5 is the mean of that of specimens 1 and 2. That of specimen 3, on the other hand, bears no relation to any of the others. Its location in the sheet was well outside the limits of the parts of 1 and 2 which were ex-

amined, and the actual structural appearance of this part was quite different from that at the center. The surface was darkened with oxide and seemed more close-grained. The magnetization curves for these specimens are plotted in Fig. 3.

After specimens 1 and 3 were measured, they had been wrapped together in asbestos and annealed in a combustion furnace at a red heat. They became covered with dark oxide but showed no red, and did not seem badly burned, though the area was considerably reduced; and therefore the value to be used in computing the absolute values of the intensity of magnetization is a very uncertain quantity. The curves representing their change of length are plotted on the same sheet with the others; the magnetization curves in Fig. 4; and the values given in Tables V. and VI.

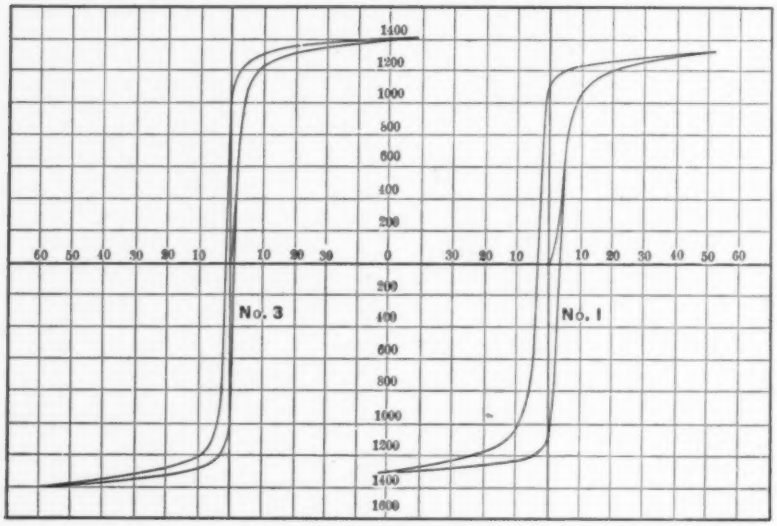

Fig. 4. Cyclic magnetization curves for Numbers 3 and 1, annealed.
Ordinates = lines of magnetization per sq. cm.
Abscissas = magnetic field c. g. s.

An increase in the retentiveness is noticeable in every curve, but the coercive force does not seem to be much affected by annealing. Specimen 3 annealed had a slight initial magnetization which was not corrected for. It is probable that this causes the whole cycle as drawn to be a little too close to the base line. From the fact

that the average of the ascending and descending curves for Nos. 1 and 3 when annealed are similar, it might seem as if annealing had removed the effects of the grain. To test this Nos. 2 and 5 were annealed. Though well wrapped up in asbestos paper and placed in a larger iron tube, they were badly burned—yet did not seem to be well annealed, for No. 2 showed the same curve as before with only a slight increase of retentiveness; while No. 5 had its maximum increase of length greatly diminished and changed so as to take place in a lower field. Consequently, it compared with No. 2 almost as No. 2 had compared with it before annealing. This

TABLE V.

Changes in length and intensity of magnetization of Specimen No. **1**, *annealed.*

H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \cdot 10^7$	H	$\frac{\Delta l}{l} \times 10^7$
2.285	0	2.285	11.50	12.43	30.29	3.20	31.39
3.20	.38	3.67	4.56	9.60	30.93	1.37	30.57
3.67	1.014	4.57	5.07	7.77	31.18	0	28.65
4.57	3.30	6.40		5.03	31.44	1.83	17.75
6.85	11.15	7.77	12.68	3.20	31.30	2.79	8.61
9.37	17.75	9.51	16.22	1.78	30.81	3.67	5.07
18.46	24.34	16.91	23.02	0	28.39	4.57	4.81
22.85	24.47	22.85	24.08	.914	23.93	4.57	5.58
31.22	23.70	31.99	23.63	11.47	30.67	5.94	8.86
56.21	18.25	56.21	18.35	7.77	31.33	7.77	13.18
32.46	24.84	32.36	25.35	4.57	31.44	9.46	16.73
22.85	27.63	20.57	28.39				

TABLE V.—*Continued.*

H	I	H	I	H	I	H	I
.82	21	34.74	1288	1.79	866	18.72	1249
1.74	74	53.47	1345	2.38	424	9.50	1222
2.38	154	32.13	1308	3.20	31	3.20	1191
3.20	286	14.22	1260	4.52	505	0	1134
4.43	555	9.50	1245	6.81	857	1.83	840
6.76	890	3.20	1214	9.45	1019	2.38	393
9.42	1058	0	1151	18.28	1178	3.20	80
14.17	1169	.86	1093	53.93	1318	4.47	562

TABLE VI.

Change in length and magnetization of Specimen No. **3**, *annealed.*

H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$	H	$\frac{\Delta l}{l} \times 10^7$
1.33	.51	4.02	5.07	9.14	27.89	6.85	27.89
2.29	5.07	5.99	12.68	6.85	27.28	3.67	27.13
4.11	12.68	9.19	18.25	3.66	24.34	1.83	24.34
6.85	19.42	16.00	22.46	1.83	20.28	.91	20.28
9.28	22.36	20.57	25.55	0	9.89	0	16.22
13.25	24.84	27.42	25.86	1.37	1.52	.91	9.89
18.28	25.81	31.53	25.76	2.29	2.03	1.46	22.05
23.58	25.86	60.78	25.35	.91	1.27	17.37	25.25
41.13	23.83	31.53	20.28	1.46	1.52	22.85	25.35
59.41	20.28	22.85	25.96	2.29	5.07	41.13	23.32
23.76	27.38	19.19	27.38	4.11	12.68		19.77
18.28	27.89	13.25	27.84	9.16			
13.71	28.04	9.37	28.09				

TABLE VI.—*Continued.*

H	I	H	I	H	I	H	I
.82	157	9.37	1364	3.10	836	9.37	1230
1.74	592	3.20	1295	6.58	1099	3.20	1162
3.10	966	0	1019	9.23	1160	0	892
4.20	1110	.46	760	26.96	1258	.46	642
9.19	1285	.82	424	52.10	1333	.82	287
23.58	1384	1.38	21	60.78	1350	1.38	176
58.50	1470	1.74	370	52.10	1336	1.74	521
23.72	1413	2.33	679	18.19	1265	2.33	829

seems to agree with Bidwell's experiments on sheet iron rings. Changes in diameter and width were observed by him for both the hard and the annealed state; and he found that the annealing had a great influence on the former, but scarcely any on the latter. Assuming that the direction in which the plates passed through the rolls was the same as the circumference of the ring into which it was made it will be seen that the difference in the effect of annealing with changes parallel to and across the field may arise from the fact that these happened to be also parallel to and across the grain. The difference due to the grain in the changes under high fields is

shown in the two curves in Fig. 5. They were taken when the specimens were unannealed.

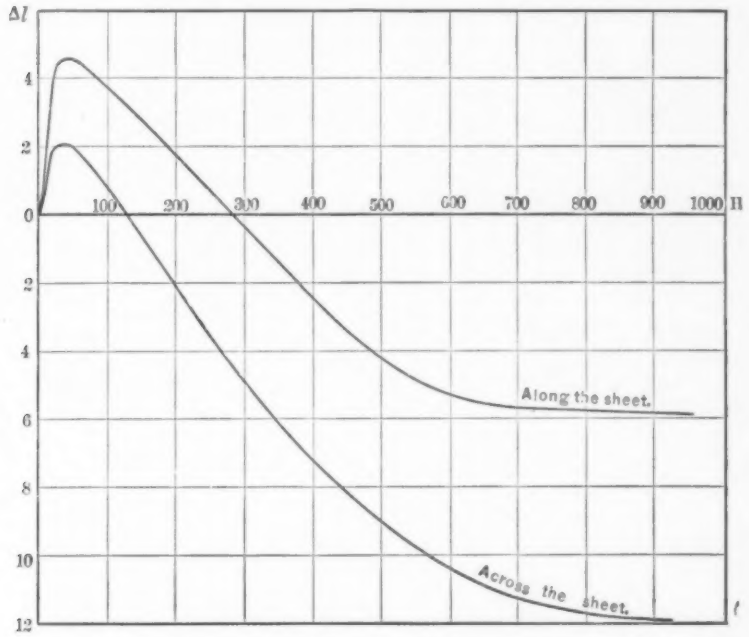

Fig. 5. Change of length curves with increasing magnetic field.
Ordinates = millionths of the length.
Abscissas = magetic field.

Young's modulus of elasticity was also measured for these two specimens and was found to be the same for both within the limit of accuracy of the measurement, about 1%. Magnetization did not seem to affect it. In dynes per sq. cm. the value was 172×10^{-11}.

It is clear from these results that the changes in volume which take place in iron wire, sheet iron, etc., when magnetized, may be due to the fibrous structure of such iron. So that it seems that we have no reason to think that an isotropic specimen would show any such change. Whether it would or not remains to be proved.

I wish here to express my thanks to Professor Rowland, who suggested this experiment to me and to Dr. Ames for his very kind interest and supervision of the work.

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A RELIABLE METHOD OF RECORDING VARIABLE CURRENT CURVES.

By J. A. Switzer.

THE following investigation was undertaken in the hope of adding something further to the work of Dr. A. C. Crehore which he published under the above title in the Physical Review (Vol II., p. 122).

The paper referred to was devoted chiefly to the description and discussion of the method proposed, and although sufficient experimental work was done to demonstrate the possibility of overcoming the many difficulties of the method, Dr. Crehore states that only "the most crude of first results" were obtained. A considerable part of the present investigation was devoted to the study of certain relations which apparently had not attracted the notice of Dr. Crehore, but which proved to be of considerable importance; indeed it is probable that without the study of these no better results would have been obtained in the present instance.

The method of recording variable current curves proposes as its salient feature the utilization of the property possessed by certain substances of rotating the plane of polarization of polarized light when placed in a magnetic field. Now the amount of rotation produced is a function of the wave-length of the light, becoming greater as the wave-length diminishes; it is also proportional to the cosine of the angle made by the direction of the lines of magnetic force with the direction of the beam of light, being therefore a maximum when these directions are coincident. Hence, if a beam of plane polarized white light be passed in the direction of lines of magnetic force through a medium having this property, there will be, on its emergence from the medium, an angular dispersion of the planes of vibration of the different wave-lengths. If then a Nicol prism, or other analyzer, be placed beyond the medium in the path of the light, certain wave-lengths will, in general, be crossed with

the analyzer and so will be refused transmission by it, and in consequence the light transmitted will cease to be white, and will have some tint, the resultant of all the wave-lengths which are not cut off by the analyzer. If, further, the light be dispersed into a spectrum, there will be a dark band in the spectrum corresponding to those wave-lengths which were refused transmission by the analyzer, Finally, the amount of rotation produced is a function of the strength of the magnetic field, being proportional to the difference in magnetic potential between the point of entrance into the given medium and that of emergence from it. Thus it follows that, having a given medium, the dark band in the spectrum will occupy one position for one particular difference of magnetic potential, and for another potential difference another position; and if the potential difference change continuously from the former to the latter value, the dark band will move continuously from the former to the latter position. Suppose the magnetic field to be that due to current flowing in a long helix of copper wire, and suppose the beam of light to pass axially through the coil, and the space within it to be occupied by the substance possessing magnetic rotatory power; then any change in the current strength, by correspondingly changing the magnetic difference of potential, immediately produces a change of the position of the dark band in the spectrum that exactly corresponds to the change in the current strength. If the spectrum thus modified be allowed to fall between the edges of a narrow horizontal slit upon a photographic plate which is moving vertically, or in a direction perpendicular to the length of the spectrum, then the motion of the dark band in the spectrum will compound with the motion of translation of the plate, and thus a curved dark line upon a field of light will be photographed upon the plate, representing with absolute accuracy the current changes which occur in the helix during the passage of the plate across the slit.

Dr. Crehore called this "a weightless vibrator," and its obvious freedom from inertia is probably not shared by any other form of vibrator in use for registering variable electric currents. Apparatus might easily be arranged in accordance with the principles thus outlined; it would consist of a source of light, a series of lenses to render the light parallel, a Nicol prism or other form of polarizer, a

tube closed at the ends by plane glass covers containing the substance possessed of magnetic rotatory power (supposing the substance to be a liquid), the coil of wire surrounding this, a second Nicol, a collimator tube with vertical slit, a dispersing prism, a photographic lens with a horizontal slit at its focus behind which falling in suitable grooves would be the holder for the photographic plate. But such an apparatus would have the serious fault that at the instant when the current value was passing through zero, since at that moment there would be no rotation of the plane of the polarized light, the vibrator would be non-existent; that is to say, the field would be either entirely dark, or else entirely lighted, according as the two Nicols were or were not crossed with each other. Hence no continuous record of an alternating or other current whose value passed through zero could be obtained. This fault can be met if between the two Nicol prisms a quartz plate, cut with faces perpendicular to the optic axis, be placed; for then there will be rotation, and therefore angular separation between the different wave-lengths, and a dark band in the spectrum, whether current flows or not, while the effect of any current changes will be, as before, to cause the band to move back and forth in the spectrum in obedience to such changes.

Thus is briefly indicated the actual arrangement of apparatus. Carbon disulphide was used to produce magnetic rotation, and the quartz plate with which work was first attempted was one 6.77 millimeters thick—the same plate which Dr. Crehore had used. But the experimental work had not proceeded far when there became apparent the necessity for investigating the relation of the thickness of the quartz plate, and the number of ampere-turns of current impressed, to the width of the dark band and its amplitude of motion in the spectrum; the desire being to obtain such proportions as should make the width the least, and the amplitude the greatest, possible.

For this investigation a simple graphical method was devised based on the following considerations:

Magnetic rotation in a given medium is represented by $\theta = vV$, in which θ = angle of rotation for any given wave-length, v, a constant for the given wave-length and given substance, called Verdet's constant, and V the difference in magnetic potential. Since

$$V = \frac{4\pi Si}{10} = 1.257Si,$$

in which S = number of turns of wire and i = current, this may be written $\theta = 1.257vSi$. Values of Verdet's constant for the solar lines B, C, D, E, F and G may be found in Landolt & Börnstein's Tables. Rotation in quartz is expressed by the formula,

$$\theta' = \frac{2\pi^2 e C}{a^4 \lambda^2},$$

in which e is the thickness of quartz, C a constant, a the velocity of light and λ the wave-length. Preston gives values for θ' for the solar lines above enumerated for a value of e = 1 millimeter.

The angular separations due to these rotations were represented by polar diagrams. Thus in Fig. 1, the upper diagram 1, shows the angular separation of the lines C, E, F, and G that would be produced by 5 mm. of quartz; the initial plane of vibration for all the wave-lengths being the line marked "zero rotation," and the planes of vibration of these wave-lengths, after emergence from the quartz being represented respectively by the lines marked "C," "E," "F," and "G." The magnetic rotation is superimposed upon that of the quartz, hence the corresponding angle of rotation of each line is laid off from the position the line has already assumed due to the quartz, in the positive direction if the magnetic rotation conspires with the quartz rotation, and in the negative if not. The second diagram corresponds to 5 mm. of quartz and 37,500 ampère-turns of magneto-motive force, the two rotations being in the same direction; the dotted lines and primed letters represent the locations of the lines after both rotations. Supposing the analyzer to be so placed that when no current flows in the coil the

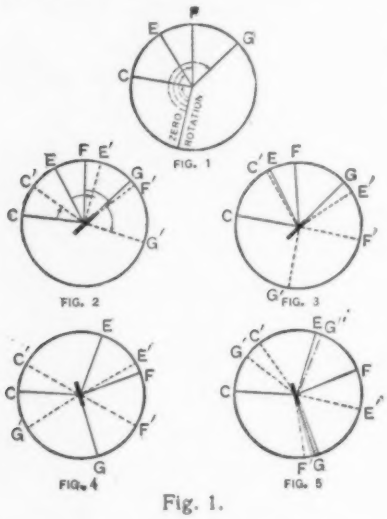

Fig. 1.

G line is crossed with it, then on the application of the 37,500 ampère-turns G moves to G', F to F', E to E', etc., so that it is no longer the G line but some line between F and E which is crossed with the analyzer. In other words the dark band moves from G to a point in the spectrum just below F. The heavy short line in the center of the diagram is intended to represent the analyzer, its direction being the direction of the plane of vibration of that light which the analyzer cuts off; so also in the succeeding diagrams. In the third diagram we have the same thickness of quartz, 5 mm., but double the magneto-motive force. Here it appears that the band moves from G to a point just below E. It becomes evident at once, therefore, that with a given thickness of quartz, increasing the magneto-motive force increase the distance in the spectrum that the dark band moves. No. 4 is the diagram for 37,500 ampère-turns and 10 mm. of quartz. Here the band will move from G only to a point a little beyond midway between G and F; we conclude, therefore, that increasing the thickness of quartz decreases the extent of motion of the band. These facts may be quantitatively expressed. Thus, for example, since the wave-length G is 4308 tenth meters and F is 4861, a motion from G to F might be expressed by the number $4861 - 4308 = 553$; or again, a motion from G to a point $\frac{10}{85}$ of the difference between E and C below E (as is the case shown in No. 3, the angle between E' and C' being 85° and that between E' and G 10°) by the number $(5270 - 4308) + (\frac{10}{85} \times 1203) = 1114$. These relative values are not accurate to a high degree, since two false assumptions are introduced, namely, that the spectrum formed by a dispersing prism is a normal spectrum, and that when the band falls between two of the solar lines, as between E and C, its position is accurately located by dividing this distance in simple proportion. Nevertheless for the purpose of this investigation the method is sufficiently exact. Reference will presently be made to the latter assumption when it will be shown in a few particular cases the exact extent of the error introduced.

It remains to inquire what effect changes in thickness and turns of wire have upon the width of band. While only a single wave-length is exactly crossed with the analyzer many wave-lengths on either side are partially cut out by the analyzer, so that the dark

band occupies a very appreciable width, and shades off gradually on either edge. We may therefore assume that all rays whose planes of vibration do not differ by more than say five degrees either way from the plane of vibration of the particular wave-length which is crossed with the analyzer will be cut out of the spectrum. Hence, supposing our dark band to be centered at *G*, we may draw radii five degrees from *G* on either side, as in diagram 5 (which is the diagram for 10 mm. quartz and 75,000 ampère-turns) to represent the width of band. The number of wave-lengths included in the band will be the greater the less the angular separation between the wave-lengths ; in other words increasing the thickness of quartz decreases the width of band. The effect of the magnetic rotation will be to increase and decrease the width as the current changes value, the band being narrowest when the magnetic rotation is a positive maximum and widest when it is a negative maximum. This variation in the width of the band will be the less the greater the thickness of quartz. It remains to be seen whether the decrease in the width of band as the thickness of quartz is increased is greater than the decrease in its amplitude of motion, as this is the consideration of paramount importance. A numerical expression for the width of band can be found in the same units as were used to express its range of motion ; thus in No. 5 the separation between the lines *F* and *G* is ninety-five degrees, these wave-lengths being respectively 4861 and 4308, the width of the band is $\frac{10}{95}$ (4861 — 4308) = 58.5.

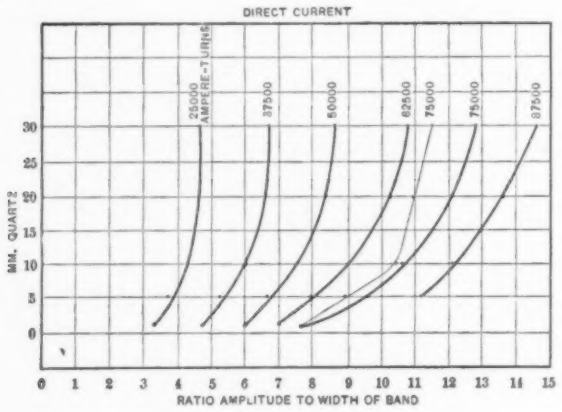

Fig. 2.

This for 10 mm. quartz. For 5 mm. the separation is half as great and so the band is twice as wide. An expression may now be obtained for the ratio of the width to the amplitude for different thicknesses of quartz and different numbers of turns of wire. Thus in the case of No. 3 (with 5 mm. and 75,000 ampère-turns) the ratio is 1114 : 117=9.52. These ratios have been obtained in this way for thicknesses of quartz of 2, 5, 10, 20, and 30 mm. and 25,000, 37,500, 50,000, 62,500, 75,000, and 87,500 ampère-turns, and the results are shown graphically in Fig. 2, in which thicknesses of quartz are ordinates and ratios abscissas. The width of band was arbitrarily assumed as 10°; changing this value would change the actual location of any given curve, but not its curvature, and inasmuch as the change in ratio with change in thickness of quartz is the consideration sought this, is immaterial. In order to exhibit the error due to the second false assumption previously spoken of—namely, that it is correct to divide the angles between the solar lines in simple proportion—in Fig. 2 is drawn an additional curve for 75,000 ampère-turns, obtained from the polar diagrams shown in Fig. 4 and the curves of rotation shown in Fig. 3 for this number

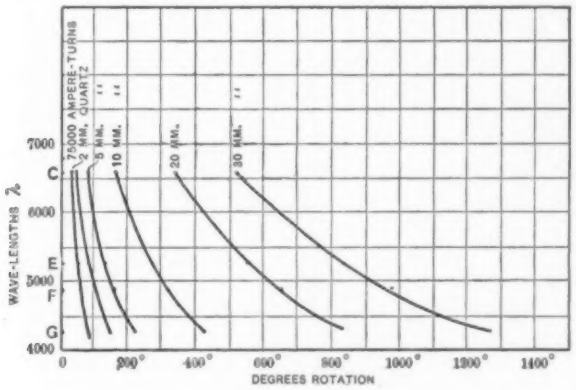

Fig. 3.

of ampère-turns and the different thicknesses of quartz. Returning to Fig. 2 we observe that for a given number of ampère-turns, the ratio increases slightly with increase of thickness of quartz, but that this increase is very much greater for the larger magneto-motive forces.

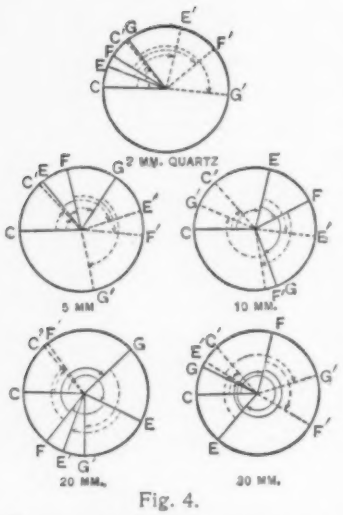

Fig. 4.
Diagrams for 75,000 ampere-turns.

The forms of these curves determined the use of a greater thickness of quartz and a coil of more turns than had been originally contemplated. Accordingly a quartz plate 20 mm. thick was procured, and a coil of 3840 turns of No. 12 wire constructed. Alternating currents of about 10 ampères (equivalent to about 20 ampères direct current in rotating power) were used. The arrangement of apparatus is shown in Fig. 5. An arc light, *a*, was used at first as the source of light, and by means of the lenses *b* and *e* and the diaphragm *c*, the light was rendered parallel. Falling on the mirror *f* it was reflected through the Nicol prism *g*, the tube of carbon disulphide *d*, the quartz plate *h*, the collimator, Nicol prism, dispersing prism and photographic lens at *m*, *n* or *p*, and finally upon the photographic plate which was arranged to fall, by gravity, in the groove behind a horizontal slit at *s*. Later sunlight was used, reflected from a heliostat represented by the dotted line near *f*, and the lenses and mirror *f* dispensed with. The arrangement for confining the carbon disulphide was found to be very efficient. The tube *a*, which was of glass and which was 152.4 cm. long and of 5.1 cm. internal diameter, had a small hole bored laterally near each end; through one of these communication was had by a bent siphon tube with a reservoir shown at *r* con-

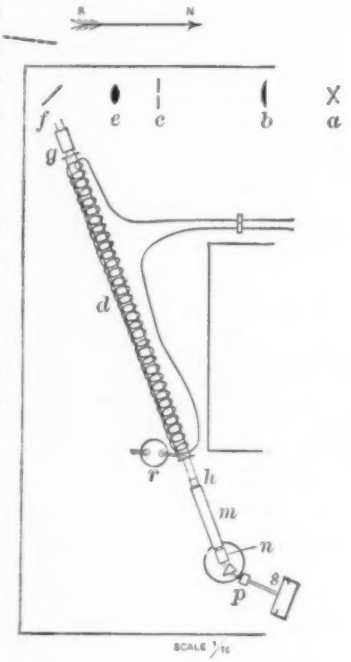

Fig. 5.

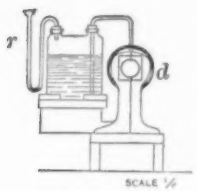

Fig. 6.

sisting of a Wolfe flask. This is shown in elevation in fig. 6. In the other hole a small straight upright tube was sealed. The tube and reservoir were then filled with the carbon disulphide, and this small tube sealed. Into the other opening of the Wolfe flask the bent funnel tube was sealed, and finally a few drops of mercury were poured into the funnel tube. This arrangement served to keep the tube completely full at all temperatures (and the apparatus was subjected to a large range in temperature); as the temperature rose and the carbon disulphide expanded, its vapor was slowly forced, bubble by bubble, past the mercury head, and as it cooled air bubbled past it into the reservoir. In this way the end plates, which were sealed in place by a soft cement, were relieved of any pressure due to expansion, and at the same time evaporation was very slow. In order to have the carbon disulphide as thoroughly transparent as possible, it was twice distilled in the presence of mercury; heat for the distillation was obtained by passing an electric current through a coil of German silver wire closely surrounding the retort.

The coil of wire surrounding the tube was wound in 6 layers, and was so arranged as to be connected in 2 or in 4 parallel circuits. With the former arrangement its inductance was .023 henrys, with the latter .0037; its resistance was 1.2 ohms. The quartz plate was a fine specimen cut for this work by Brashear. The rotation due to this thickness is such as to make three bands appear in the length of the visible spectrum.

The light from an arc lamp, after passage through so many media and after dispersion into a spectrum, is quite feeble for photographic purposes; on this account both slits, in order to get enough light to produce a photograph at all were required to be very wide. The narrowest widths that were found to give any result were about 1.5 mm. for the vertical and 6 mm. for the horizontal slit. Consequently resort was had to the use of a heliostat, and with much better results.

A few of the photographs obtained by the method above described are reproduced in the accompanying plate.

Fig. 7 represents about the best result that was obtainable with the arc lamp as source of light. This curve is poorly defined, owing to the large openings of the slits. It was obtained when using current from a Sprague alternator, at a periodicity of about 25 alternations per second. About 15 ampères of current were flowing ; the coil was connected in four parallel circuits and dispersion was produced by a carbon disulphide prism.

For the remaining photographs sunlight was used; the coil was connected in two parallel circuits and a crown glass prism produced the dispersion. Curves 8 and 9 were obtained using current at a periodicity of 105 and an e. m. f. of 220 volts, the current being obtained from four transformers in series. For these curves the width of the horizontal slit had been reduced to 2 mm., but the opening of the collimator slit was the same as before, 1.5 mm. Curve 8 was obtained with the plate falling more rapidly than curve 9. The last curve, Fig. 10, was obtained when using current from a Smith eight-pole generator, at about 75 periods per second. For this the opening of the vertical slit had been reduced to 0.5 mm., the horizontal remaining at 2 mm.

These photographs are selected from many as being the best that were obtained. That results very much better than these could be obtained by this method, the writer doubts ; and, while the method presents itself as one full of attractiveness, its limitations are necessarily so great that it is not likely to find practical application—at least to the recording of variable electric currents. Dr. Crehore has however applied the principle, with signal success, to quite another problem, that of recording the velocities of projectiles ; and it may be that still other fields of usefulness are yet awaiting magnetic rotation of polarized light.

The work above described was carried on in the Physical Laboratory of Cornell University during the spring of 1896. A number of causes have combined to prevent its earlier publication.

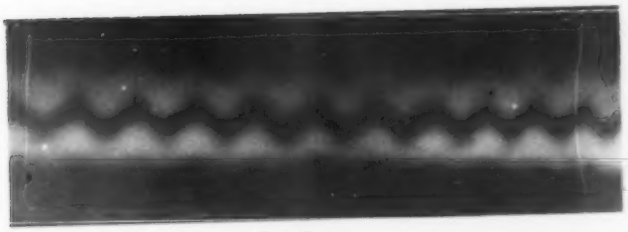

FIG. 7.

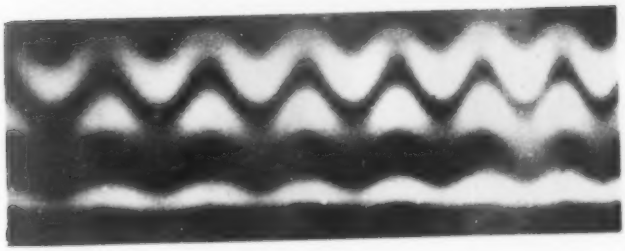

FIG. 8.

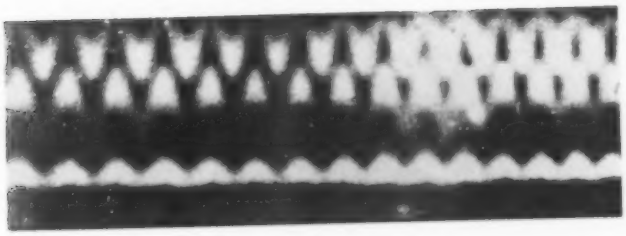

FIG. 9.

FIG. 10.

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THE PHOTOGRAPHY OF MANOMETRIC FLAMES.

By Edward L. Nichols and Ernest Merritt.

THE manometric flame has long been recognized as affording one of the most important and fascinating methods for the study of sound phenomena. Unfortunately its use has heretofore been largely confined to work of a qualitative or illustrative character. In view of the difficulty of accurately measuring the rapidly changing images that appear in a revolving mirror it is not strange that this should be true. In recent years, however, it has been found possible to photograph the vibrations of the flame upon a rapidly moving plate; and a permanent record is thus obtained which may be studied at leisure. Under favorable conditions the negatives are so sharp that measurements may be made with a high degree of accuracy. As a means of determining pitch, the manometric flame, used in connection with photographic methods, already has important quantitative applications.

For purposes of illustration the manometric flame has been chiefly used in the study of sound quality. The appearance of the flame in a revolving mirror gives so obvious an indication of the simplicity or complexity of the vibrations in question, that a better form of apparatus for lecture purposes can scarcely be imagined. In many cases it is possible to determine by inspection what overtones are present, and even to form an estimate of their relative intensities. This is made easier by first photographing the vibrating flame; several vowel sounds have been studied in this way with satisfactory results. The analysis made by means of the manometric flame is not quite complete, however, for the intensities of the various component vibrations can never be exactly determined.

The work described in the present paper was undertaken in order to determine to what extent the manometric flame, with the aid of photographic methods, could be made useful in the study of the human voice. It was decided first of all to study whole words and

sentences rather than mere isolated sounds; for it is at least questionable whether a continuously sounded vowel has the same character as it would have if spoken naturally as a part of some word. It is clear that whole words cannot be studied without the aid of photography; and it is equally clear that some special form of apparatus is necessary, in order that photographs may be obtained conveniently and with certainty. Attempts to photograph whole words upon a sliding plate were usually unsuccessful, even when ten inch plates were used. Long films were finally employed, these being mounted upon a continuously moving drum. The main features of the apparatus used are described below.

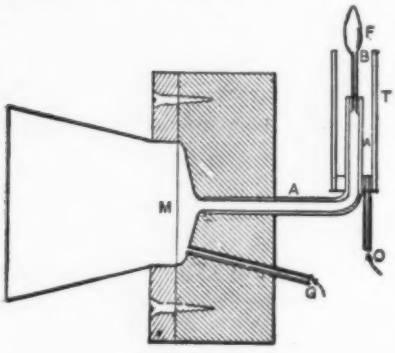


Fig. 1.

In order to obtain a sufficiently brilliant flame to make photography both possible and easy, a burner was used in which the gas jet was surrounded by an atmosphere of oxygen. A diagram of the burner and manometric capsule is shown in Fig. 1. This form of burner differs from those ordinarily used only in the presence of the outer tube *T*, which surrounds the jet proper and extends as high as the bottom of the flame. Gas enters the manometric capsule through the tube *G*, and proceeds through the short brass tubing *AA* to the platinum tip *B*. Oxygen enters the outer tube at *O*. The flame is thus completely surrounded by oxygen, and burns with an intensely white light whose actinic power is far greater than that of an ordinary gas flame. The sensitiveness of the flame to changes of pressure is even greater than that attained with the usual form of jet. This method of photographing the manometric flame has already been described by one of the writers.[1] Even ordinary illuminating gas, when burning in an atmosphere of oxygen, gives so brilliant a flame that sharp negatives can be obtained. The actinic effect of the flame can be greatly increased by enriching the gas with benzene or some other hydrocarbon. Even under these cir-

[1] Merritt: PHYSICAL REVIEW 1, p. 166, 1893.

cumstances, however, a certain amount of care is necessary in the adjustment of the flame, and in the development of the negatives, in order to be sure of obtaining sharp records. In the present investigation we therefore used instead of illuminating gas a mixture of acetylene and hydrogen. In this case the brilliancy of the flame was such that it was almost painful to the eye, and the actinic effect was so great that the exposure was sufficient even in the case of the most rapid motion of the photographic films. With pure acetylene the flame was still more brilliant; but with the pure gas it was difficult to prevent the formation of soot, which, by its accumulation at the mouth of the burner, destroyed the symmetry of the flame and caused considerable annoyance. When the acetylene was diluted with an equal volume of hydrogen this trouble was removed and the brilliancy was still quite sufficient. Instead of hydrogen, ordinary illuminating gas may be mixed with the acetylene with the same results.

In the study of continuously varying sounds, such as complete spoken words or sentences, it was necessary to obtain a photographic record which should extend over a second or more. A wide separation of the successive images of the flame was also desirable in order to bring out clearly all the characteristics of the vibrations. For these reasons the photographs were taken upon long celluloid films instead of upon plates. The films used in the final experiments were 48 in. long and 3 in. wide. They were mounted upon the cylindrical surface of a large drum of tinned iron. The width of the drum was the same as that of the film, while the circumference was slightly less than 48 in. The film therefore extended completely around the drum, lapping over slightly at the ends. It was held in place by little clips of spring brass placed at intervals of about six inches along the edges of the cylinder. The drum, which was mounted upon a shaft in the interior of a light-tight box, could be driven at any desired speed by an electric motor. By means of the device described below the exposure could be made to begin at any time desired and to automatically cease at the end of one revolution of the drum.

The general arrangement of the apparatus is shown in Fig. 2. The flame and manometric capsule being at *F*, the camera was

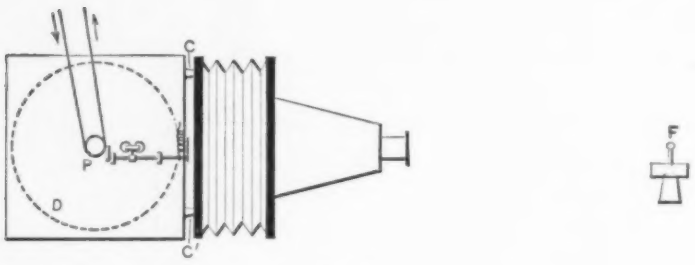


Fig. 2.

placed at such a distance that the image was very nearly of the same size as the flame itself. The rectangular box containing the drum D, on which the film was mounted, was placed just back of the camera, and the image of the flame fell upon the film after passing through a small window in the box. Two corks C and C', fastened to the box, made it possible to replace the latter, after it had been removed in order to develop a film, in the proper position to secure a sharp focus.

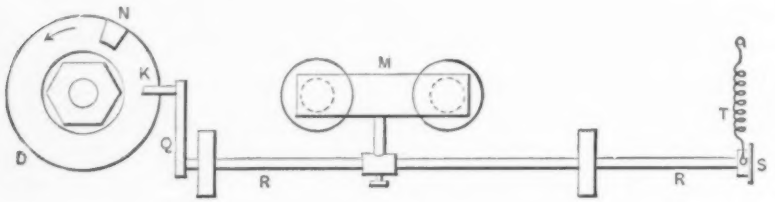


Fig. 3.

The device for starting and stopping the exposure, shown on a small scale in Fig. 2, will be better understood from Figs. 3, 4, and 5. The shutter S (see Fig. 5 and Fig. 3) was mounted at the end of a rod RR (Fig. 3), extending from one edge of the film box toward the axis of the revolving drum. The narrow window W was usually completely covered by the shutter; but by turning the rod RR through a small angle the shutter could be moved into the position shown in Fig. 5, and the film back of W was exposed. At the end of the rod RR was attached an arm Q about an inch long. This carried a little pin (K, Fig. 3). Except during the time of an exposure this pin rested in a small notch or groove (N,

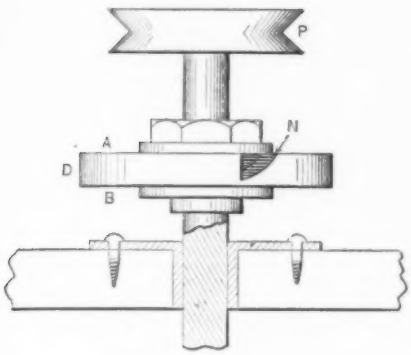


Fig. 4.

Figs. 3 and 4) in the disk D and prevented the latter from revolving with the drum. As shown in Fig. 4, the disk D was supported on the same shaft that carried the drum, but except for the friction of the washers A and B, there was nothing to compel it to revolve with the shaft. The pressure was adjusted so that D was held still when the pin K was in its notch N without retarding the motion of the drum; but as soon as K was raised out of the way the friction of A and B caused the disk to rotate.

The shutter was opened by means of the electromagnet M operated by the experimenter. Upon exciting the magnet the armature I was raised to the position shown in Fig. 5. I being connected by means of a short arm with the rod RR, the latter was thus turned sufficiently to open the shutter. At the same time the pin K was raised out of its notch and the disk D being now free to move began to revolve with the drum. So long as K rested upon the upper surface of the disk it was impossible for the shutter to close. But when one complete revolution had taken place, N was brought back again to its starting point, the spring S caused the pin to drop back into its place, and the shutter closed. The experimenter thus had it in his power to start the exposure at any time desired, and the shutter automatically closed at the end of one revolution.

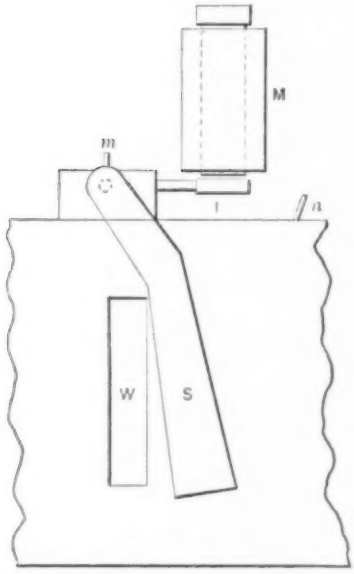


Fig. 5.

By means of the apparatus just described numerous photographs were taken for the purpose of illustrating the applications of the method and of determining the range of its usefulness. It is possible to reproduce only a few of these here and we have selected those which show most clearly the character of the results which may be obtained with the acetylene flame in oxygen and the revolving drum camera.

A large number of exposures were made in the study of combinations involving an initial consonant such as *b*, *p*, *t* and *d*, with various vowel sounds. It was found that the manometric flame underwent oscillations, during the short intervals of time required for the mouth to open and for the organs to assume the positions requisite for the utterance of the full vowel sound, which may fairly be said to be characteristic of the consonant in question. The grouping of the flame images was not so persistent as in the case of the well-known groupings which correspond to the vowel forms. While there seemed to be a certain persistence of type, the forms for each consonant were found to vary according to the vowel which followed, and likewise with the stress or emphasis placed upon the syllable. The same combination uttered by different speakers showed marked differences but without departure from the type. So far as these records are concerned, however, the initial consonants might be considered as merely slightly different ways of opening the mouth in preparation for the utterance of a vowel. The little sybillant and explosive noises and other sounds by means of which we distinguish the various consonants, produce but little effect upon the manometric flame unless they are exaggerated or reinforced unduly by an effort of the speaker.

The uncertain features by which we must distinguish between consonants in the flame records are not, as a rule, produced by the vibrations which serve us through the aural sense.

The records labelled *die* and *sigh* in Plate I,[1] show the vibrations undergone during the first tenth of a second when those monosyllabic words are pronounced. It will be seen how insignificant are

[1] It is to be observed that the records must be read *from right to left*, instead of from left to right. This reversal was made necessary by the method of reproduction and in preparing the plates.

the differences between the two records. The hissing sound of the *s*, which precedes the vowel in the latter word, although perceptible to the ear, makes no very definite record upon the film, when the velocity is one meter per second. There are indications of a disturbance of the flame during this brief preliminary period which would, perhaps, take on some well-defined form at higher speeds. The same faint indications of what might be developed into a definite form may be noted in the beginnings of the records *say* and *spray*, Plate I. In the latter case, where the motions of the mouth previous to the vowel are more complex, the consonant record assumes a more interesting character. In the record *three*, Plate I, the effect of the consonant is still more noticeable and we have indications of the characteristic form of the rolling *r* which is shown in its full development in the word *raritan*, Plate III, and in more detail in the high speed record of the rolling *r* in Plate II. In the last-named case the velocity of the drum was about 5 meters per second.

A study of various negatives, made with a view to the comparison of the records of a given consonant followed by different vowel sounds, showed such variations as to raise the question whether consonant records could be said to possess any useful significance whatever. Compare for example, *d* in *die* and *doctor*, Plate I, with the beginnings of *dah* and *doe*, Plate II. Also *s* in *sigh* and *say*, Plate, I, *t* in *to* and in *tor* of *doctor*, Plate I, with *t* in *terous* of *preposterous*, Plate III, likewise *p* in *pre* and in *pos* of that word, Plate III.

Records made at different times for the same voice with the same combinations were found, however, to show a remarkable coincidence which often extended to the minutest details. See in illustration of this the syllables *dah* (I) and *dah* (II), Plate II.

What we have called the characteristic record of a consonant was found to be best developed in accented syllables and more or less fully suppressed in those which were unaccented. Compare for example, the *p* in *pre* and in *pos* of *prepósterous*, where the accent was placed strongly upon the antepenult.

The first four records of Plate II give the full history of a single syllable deliberately spoken. The duration in such cases is about

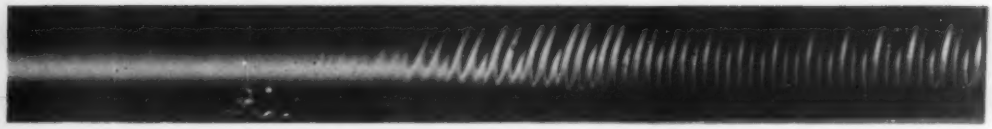
three-tenths of a second. In ordinary continuous speech, monosyllabic words upon which stress is not laid, and unaccented syllables of longer words are of much shorter duration. See *in* and *to*, also *tor* of *doctor* and *nig* of *Koenig*, in Plate I. Often, indeed, the unaccented portions of a phrase are suppressed to such an extent that they scarcely affect the manometric flame. See, for example, the last two syllables of *preposterous*, and the *i* in *raritan*, Plate III. The final syllables of words ending in *open-mouth* sounds are, on the other hand, often greatly prolonged and the record fades away by insensible gradations. See *ach*, Plate I, and *river*, Plate III.

The varied sounds which we represent in written language by the symbol *r*, are well illustrated in the photographs contained in Plates I, II, and III. We have the rolling *r* in *raritan*, Plate III, the smooth initial *r* in *river*, Plate III, in the production of which care was taken to avoid any tremulous motion of the tongue ; an intermediate form in *three*, Plate I, suppressed forms in p*r*eposte*r*ous, Plate III, and *spray*, Plate I, the curtailed final form in *doctor*, Plate I, and the prolonged final form, to which reference has already been made, in *river*, Plate III.

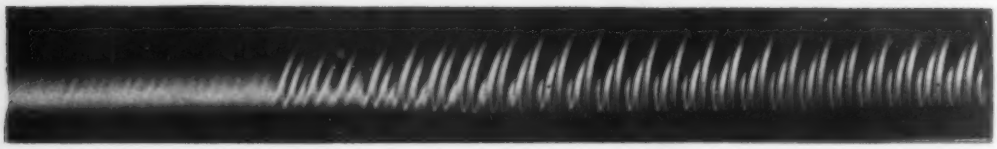
Another point of some interest which we were able to establish by inspection of the original films, although it is not brought out in the plates for the reason that these have of necessity been made up of fragments cut from the former, may be stated as follows : *The pauses between words in ordinary speech are not of greater duration than the pauses between syllables.* To an observer who tries to read a manometric film there is no way of separating words from one another until the record has been interpreted and each group of flame-images corresponding to a syllable has been assigned its place in the phrase. The reader's position is akin to that of him who first listens to a foreign tongue.

It is obvious from the illustrations which accompany this paper that the over-tones of high frequeney to which the character of articulate speech is largely due are for the most part absent from the records obtained by the method which we have used. There are, however, occasional indications of the presence of other vibrational disturbances of the flame than those which constitute the ordinary manometric record. These are distinctly apparent in the

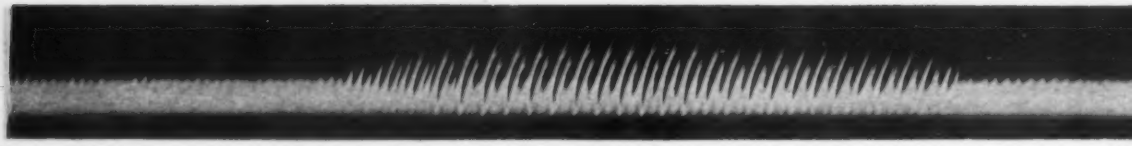
(To face page 100)



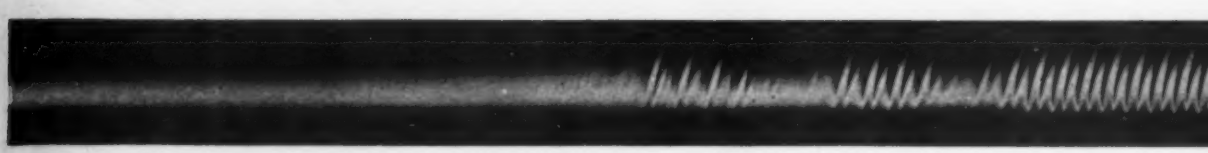
Die



Say

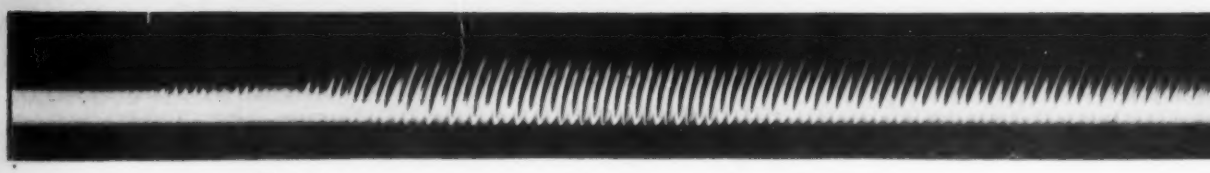


In



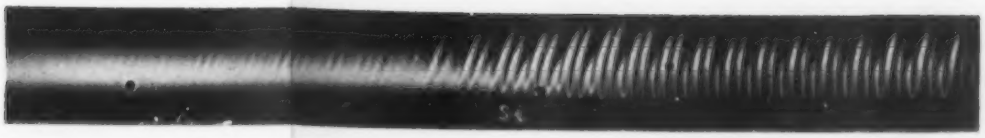
Three

Doctor

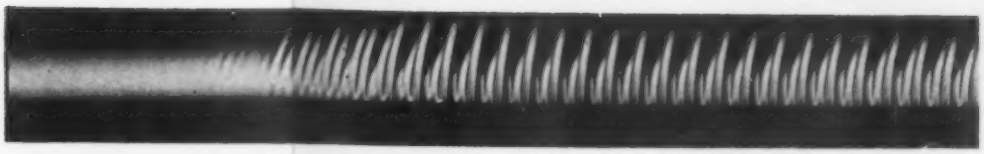


Koenig

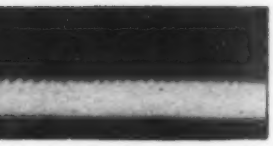
PLATE I.

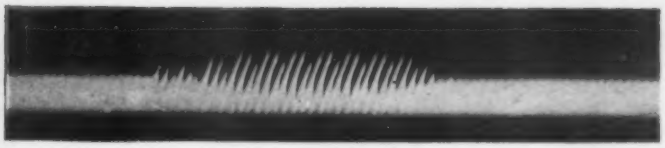


Sigh

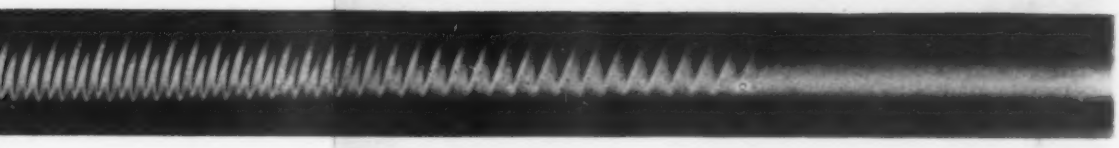


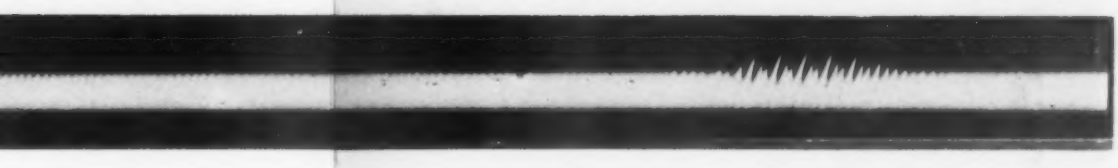
Spray

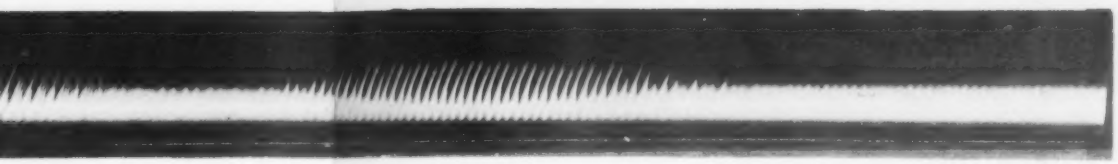




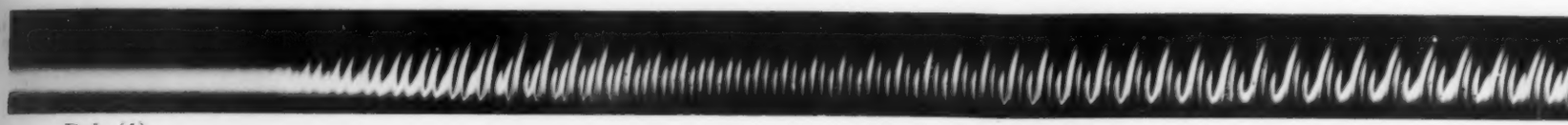
To







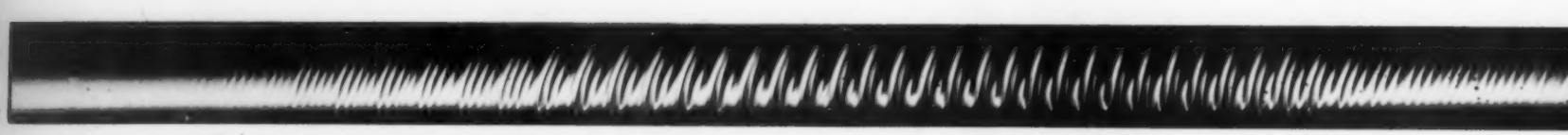
PHY OF MANOMETRIC FLAMES.



Dah (I)

Dah (II)

Doe

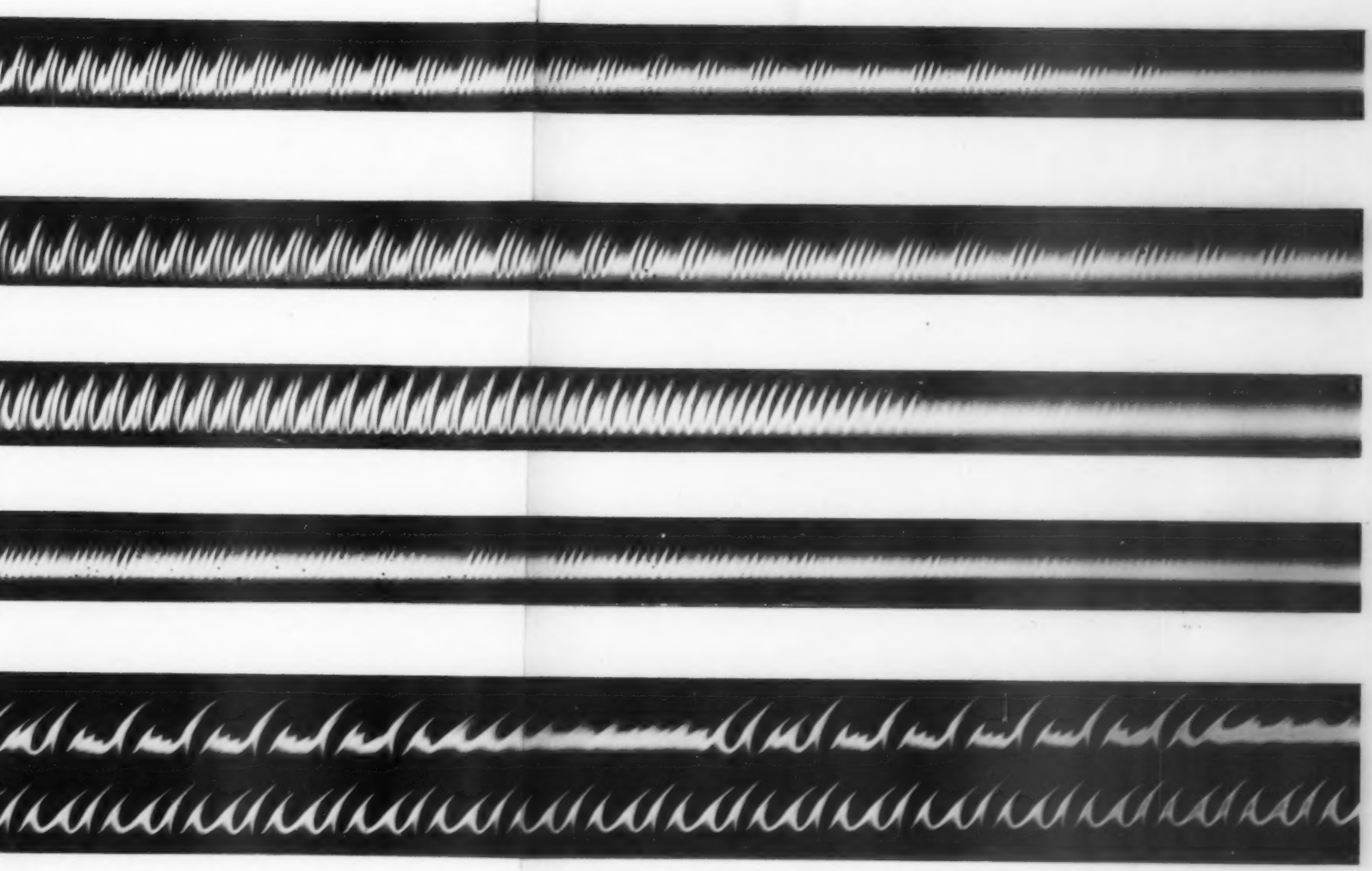


Ach

R (rolling)

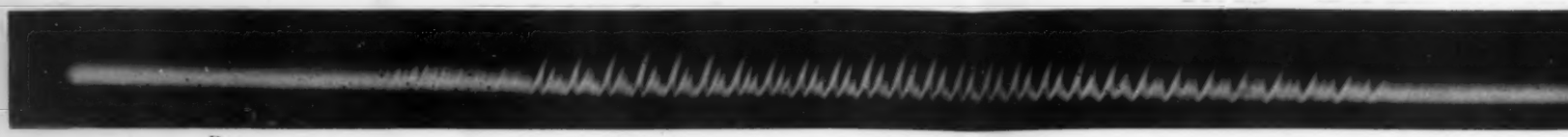
A (A flat)

PLATE II

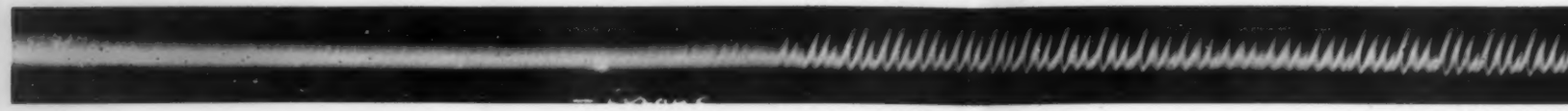


RAPHY OF MANOMETRIC FLAMES.

UNIV. OF MICH.

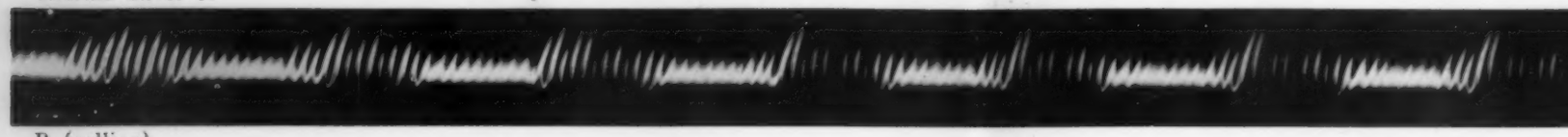


Pre



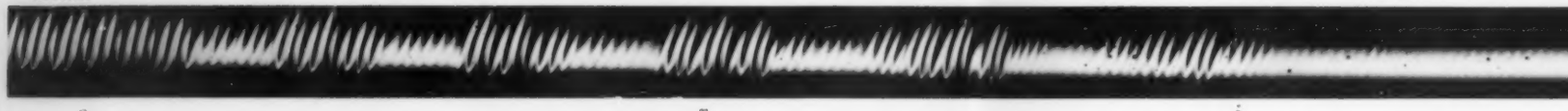
—terous

Raritan River N



R (rolling)

aritan

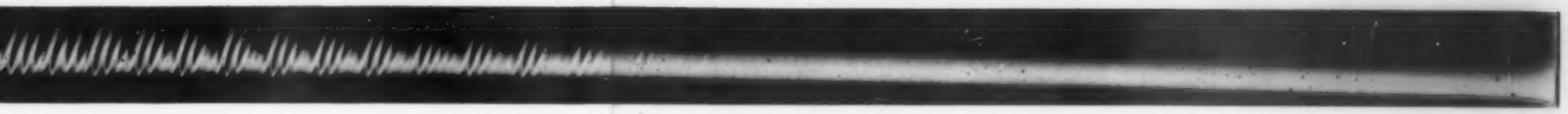


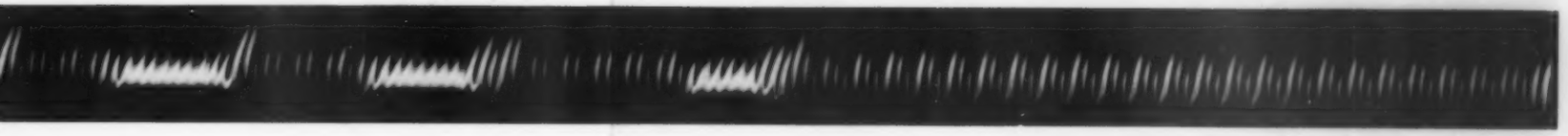
a r i

River

Invent

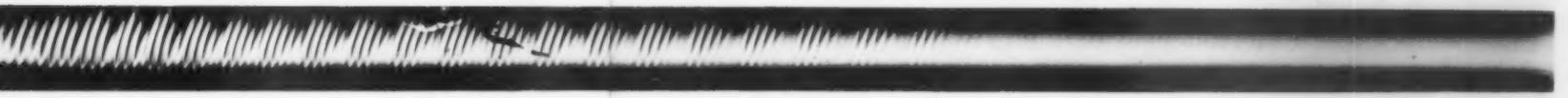
Pos

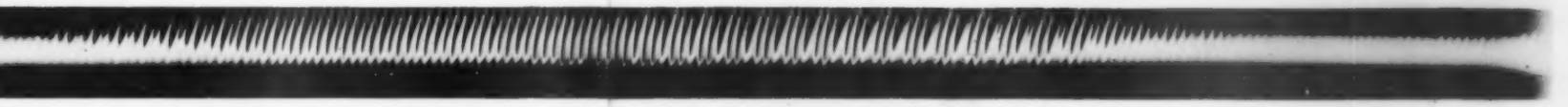




a

tan





RAPHY OF MANOMETRIC FLAMES.

negatives from which *doctor* and *Koenig*, Plate I, are taken. In the high speed record of the rolling *r*, Plate II, of which only a small portion covering in all about one-twentieth of a second has been reproduced there is further evidence of these finer vibrations. It seems likely indeed that much might be learned by working at higher speeds than we have attempted in the present investigation, the purpose of which did not extend beyond the demonstration of the practicability of applying photography, by means of the acetylene-in-oxygen flame, to Koenig's beautiful manometric methods for the study of sound vibrations.

ON THE PERIOD OF A WIRE VIBRATING IN A LIQUID.

BY LIZZIE REBECCA LAIRD.

PHYSIOLOGISTS often explain the operation of hearing, in part, on the assumption that certain very short fibers, immersed in a liquid in the inner ear, are set in vibration by resonance. This suggested the following investigation on the change of the pitch of a fine wire vibrating in a liquid medium.

Du Buat[1] was the first to conclude that a quantity of fluid is dragged with a moving body; later Bessel[2] reached the same conclusion independently, and proved it with spherical pendulums oscillating in water. Baily[3] further found conclusive proof from his pendulum experiments. Ch. Montigny[4] experimented on the change of pitch of bells immersed in water and other liquids, and found the interval to be from 1.1 to 1.5; the pitch of a tuning-fork also was graver the denser the liquid. F. Auerbach[5] determined the lowering of pitch of four tuning-forks in water, and found the interval about 1.12. Kolacek[6] found that the lowering in sulphuric acid and mercury agreed with Auerbach's results, supposing the forks weighted by some constant multiple of the density of the fluid. Later Auerbach[7] experimented on the pitch of glass cylinders filled with water, and found that the intervals varied from 1.36 to 1.75, were smaller the higher the pitch of the empty glass, were not noticeably dependent on the height, and were greater the narrower the cylinder; also that they increased with the density of the liquid according to a law.

The method of the experiment here described was to drive a wire by means of an electromagnet, as a tuning-fork is driven. For this purpose a pianoforte steel wire .446 mm. diameter was used. Contact was made and broken by a piece of fine platinum wire, 23 mm. long and .22 mm. in diameter, which was bound on to the steel wire with some fine German silver wire. One free end of the platinum dipped permanently in a mercury cup just beneath the wire, the other reached above the surface of the liquid and made contact with the mercury in an upper cup connected with a reservoir and plunger. This platinum wire was about 20 cm. from one end of the vibrating portion, 107 cm., of the steel wire and weighed

[1] "Principes d' Hydraulique," 1786.
[2] Abhandlungen d. Akad. d. Wissenschaften, Berlin, 1826.
[3] Phil. Trans., 1832.
[4] Mémoires de l'Acad. de Belg. (2) 50, p. 158.
[5] Wied. Ann., Bd. 3, 1878.
[6] Wied. Ann., Bd. 7, 1879.
[7] Wied. Ann., Bd. 17, 1882.

not more than one-sixtieth of it ; so that the effect of this loading on the period would not be more than .6 per cent. (See Donkin's "Acoustics," p. 147.) The steel wire was stretched in a metal box 109 cm. long, and was more than 2 cm. distant from the walls. The tension was measured by a horizontal spiral spring 44.7 cm. long, stretched by weights attached to it by a string passing over a pulley. After the tension was adjusted, the wire was clamped between scissor clamps, rigidly screwed to the box, so that the tension remained constant, and the vibrating portion was of a definite length. In circuit with the contact breaker, and the driving electromagnet, were the battery, and a recording magnet writing on a revolving drum, upon which the vibrations of a standard 64-fork were also recorded for comparison.

Preliminary experiments were made with shorter and thicker wires, and with the contact breaker at various points along the wire, in order to find the best conditions. The chief sources of error were found to be the magnitude and duration of the pull of the electromagnet. If the magnet was placed too near the wire the period was sensibly decreased. This effect was more noticeable in water than in air, and was as much as a change of 3 vibrations in 90, when the distance of the magnet from the wire was changed from 15 mm. to 3 mm. The level of the mercury in the upper cup influenced the duration of the pull of the magnet, and so similarly effected the period. In all cases the driving magnet was placed as far off, and the contact made as short, as possible for the maintenance of vibrations, so that the error introduced was not greater than the mean variations of the readings. In the case of the potassium carbonate solutions, trouble was experienced from the creeping of the liquid into the upper cup, where it gave rise to curious ripples on the mercury surface, which at times entirely vitiated the results.

The following table gives in the first column under each tension the observed average vibration frequencies.

Tension in grams.	2980			4760			6240			7180			8900 (?)		
Medium.	Obs.	Cal.	Δ	Obs.	Cal.	Δ	Obs.	Cal.	Δ	Obs.	Cal.	Δ	Obs.	Cal.	Δ
Air	73.8			92.3			105.5			113.4			126.5		
Water	70.1	67.2	2.9	87.9	84.3	3.6	99.4	96.6	2.8	107.7	103.9	3.8	117.8	116.0	1.8
K_2CO_3 Solution $\rho = 1.47$	65.1	64.3	.8	82.6	80.9	1.7	93.9	92.7	1.2	99.7	99.8	−.1	111.6	111.4	.2
do. $\rho = 1.22$	67.0	65.8	1.2	84.4	82.6	1.8	95.4	94.7	.7	105.5	102.0	3.5			
Mercury	43.0	41.2	1.8	53.3	52.0	1.3	61.9	59.7	2.2	65.8	64.3	1.5	73.4	71.9	1.5

It is interesting to compare these results with the mathematical theory as worked out by Stokes (see Camb. Phil. Trans. Vol. IX., pt. 2, p. 35), for a cylinder oscillating perpendicularly to its axis, where it is assumed that the resistance on any element is the same as if it belonged to an infinite cylinder oscillating with the same linear velocity, and so that the square of the velocity may be neglected. The resistance per unit length is

$$M' \left\{ 1 - \frac{4F'(a)}{m^2 a F(a)} \right\} \frac{dV}{dt}$$

where M' = mass of displaced fluid, V = velocity of cylinder, a = radius,

$$m^2 = \frac{2\sqrt{-1}\pi\rho}{\tau\mu},$$

where ρ = density and μ = viscosity of the fluid, τ = period of vibration, and $F(a) = u_{r=a}$ where u is given by

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - m^2u = 0$$

subject to the boundary conditions. The above table gives the values calculated on this theory in the second column under each tension, and the differences between these and the observed values in the third. For water the observed values agree much better with numbers calculated on the simple supposition that the wire is loaded with the mass of the displaced fluid.

To show the amount of variation in the readings taken at different times the set for water is given below. In the case of air this variation was less than 1%.

WATER.

2980		4760		6240		7180		8900 (?)	
	Error		Error		Error		Error		Error
70.6	.5	85.5	—2.4	99.5	.1	109.4	1.7	117.6	—.2
69.7	—.4	85.6	—2.3	99.6	.2	107.1	— .6	117.3	—.5
70.6	.5	87.1	— .8	100.6	1.2	109.0	1.3	118.2	.4
70.0	—.1	88.7	.8	99.1	—.3	107.2	— .5	118.2	.4
69.8	—.3	88.6	.7	98.8	—.4	106.0	—1.7		
		89.8	1.9	99.0	—.6				
		90.0	2.1						
		87.6	— .3						
		88.3	.4						
		87.6	— .3						

Attempts were made to perform the experiments in oil and in glycerine, but though it was possible to drive the wire, the vibrations as recorded were very irregular. This was supposed to be due to eddies. It is in-

tended to try the experiment with stouter wires and bars, where eddies will not be as effective. In glycerine the vibration frequency was diminished from 73.8 in air to values ranging from 15 to 22 per second. Calculation from the theory gives 21. The agreement between theory and experiment is as close as could be expected, considering that the theory takes no account of eddies, nor slipping at the surface, and supposes the velocity small.

Although the experiments made with the wire 37 cm. long and .933 mm. diameter, were not performed under very satisfactory conditions, some of the results may be recorded as also showing approximate agreement with the theory.

Tension.	8 Kgm.			10 Kgm.		
Medium.	Obs.	Cal.	Δ	Obs.	Cal.	Δ
Air	141.9			157.4		
Water	132.3	131.8	.5	148.6	146.4	2.2
Oil	120.3	119.9	.4	133.9	133.7	.2
Sodium Nitrate Solution ρ=1.38	126.7	128.6	−1.9	140.2	142.8	−2.6

For suggestions and assistance the writer wishes to express her thanks to Professor A. S. Mackenzie, under whose direction this investigation was carried out.

PHYSICAL LABORATORY, BRYN MAWR COLLEGE,
May, 1898.

A SIMPLE MECHANICAL ILLUSTRATION OF APPARENT POTENTIAL ENERGY WHICH IS REALLY KINETIC.

BY ERNEST MERRITT.

IN recent years the attempt to bring all branches of physics into obedience to the fundamental laws of mechanics has been quite popular. In many cases the attempt is in large part successful; and the employment of generalized coördinates makes it possible to discuss almost all branches of physics and show that they conform to mechanical principles, even when the details of the mechanical processes involved are not known. A difficulty sometimes arises, however, in attempts of this kind on account of the impossibility of distinguishing between potential and kinetic energy. In the case of the magnetic and electrostatic fields, for example, if we think of the field as being in a state of strain, we naturally look upon the energy as potential; whereas if we consider the field as due to some condition of motion in the ether, it is proper to think of its energy as kinetic. So long as we do not know what really goes on in the field it is impossible to decide which of these two views is correct. In this case, as in many other similar ones, it is possible to discuss the problem by the aid of generalized coördinates, and to obtain correct results, no matter which hypothesis is made regarding the character of the energy. The matter is much simplified, however, if we accept the view that in reality *all* energy is kinetic, whether it be the energy of moving bodies of finite size, or the energy of some motion whose existence we can only imagine.

According to this hypothesis the energy due to a strained condition, such for example as that of a stretched spring, would in reality be the energy of some more or less complicated motion of the ether; this motion would be looked upon as the cause of the strain, and in fact as constituting the strained condition. If work is done against gravity in lifting a weight, we usually speak of this work as being stored in the form of potential energy; yet it is equally plausible for us to say that the work that we do in lifting the weight reappears as the kinetic energy of something analogous to eddies in the ether, these being brought into existence by the movement. This motion will continue as long as the body occupies its new position, and may be the cause of the force tending to make it fall. When the body is allowed to fall the energy stored in this motion of the ether is given back again and appears as visible motion.

The view that all energy is kinetic was advanced a number of years

ago, and it has been shown in several ways that it is possible to explain the existence of apparent potential energy upon this hypothesis if certain plausible assumptions are made regarding the character of the motion in the ether which corresponds to it. A general statement of the matter is as follows:

Consider a mechanical system, in general very complex, in which the energy is in reality all kinetic. If the position of every particle in the system is completely specified by the generalized coördinates $\varphi_1, \varphi_2, \cdots$ the kinetic energy T will be

$$T = f(\varphi_1, \varphi_2, \cdots, \dot{\varphi}_1, \dot{\varphi}_2, \cdots) \qquad (1)$$

where f is homogeneous and quadratic as regards the generalized velocities $\dot{\varphi}_1, \dot{\varphi}_2, \cdots$. If Φ_n is the generalized force acting to increase φ_n the equations of motion will take the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_n} - \frac{\partial T}{\partial \varphi_n} = \Phi_n. \qquad (2)$$

Assume now that the existence of certain essential coördinates is unknown to an observer who is studying the system in question. Let these unknown or "concealed" coördinates be denoted by $\psi_1, \psi_2, \cdots$. If certain restrictions are placed upon these coördinates and the motions corresponding to them, they may be made to explain the existence of apparent potential energy.

The necessary restrictions are: (1) that the concealed coördinates enter the expression for T only as velocities, *i. e.*, the $\dot{\psi}$'s appear in this expression, but not the ψ's themselves; (2) no outside forces act to change the concealed coördinates; (3) terms of the form $\dot{\varphi}_m \dot{\psi}_n$ do not appear in the expression for T. When these assumptions are made the expression for the kinetic energy may be separated into two parts such that

$$T = f_1(\varphi_1\varphi_2 \cdots \dot{\varphi}_1, \dot{\varphi}_2, \cdots) + f_2(\varphi_1\varphi_2 \cdots \dot{\psi}_1\dot{\psi}_2 \cdots) \qquad (3)$$

f_1 contains only the known coördinates and their time derivations, while f_2 contains the known coördinates and the velocities of the concealed coördinates. Since no outside forces act to change the velocities of the concealed coördinates, the values of $\dot{\psi}_1, \dot{\psi}_2$, etc. will depend only upon the configuration of the system, *i. e.*, each of the $\dot{\psi}$'s is a function of the known coördinates φ_1, φ_2, etc.[1] f_2 is therefore in reality a function of the known coördinates only.

[1] Analytically this is shown as follows: The equation of motion for each of the concealed coördinates will take the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\psi}} = \frac{d}{dt}\frac{\partial f_2}{\partial \dot{\psi}} = \Psi = 0$$

An observer who was ignorant of the existence of the concealed coördinates would naturally look upon f_2 as the expression for the *potential energy* of the system, for when the ψ's have been eliminated f_2 will appear to be a function of position only. f_1, on the other hand, being homogeneous and quadratic in the $\dot{\varphi}$'s, would doubtless be accepted as the complete expression for the kinetic energy. This assumption of potential energy, when in reality none exists, would not merely be extremely plausible, but its incorrectness would also be difficult of proof; for all predictions based upon it regarding the behavior of the system would be correct so far as they could be experimentally tested. Consider, for example, the equation of motion for one of the known coördinates φ. Since the system possesses no potential energy the equation will take the form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}} - \frac{\partial T}{\partial \varphi} = \Phi \tag{4}$$

where Φ is the generalized force acting from outside the system to increase φ. As T consists of the two parts f_1 and f_2 the equation may be written

$$\frac{d}{dt}\frac{\partial f_1}{\partial \dot{\varphi}} - \frac{\partial f_1}{\partial \varphi} - \frac{\partial f_2}{\partial \varphi} = \Phi \tag{5}$$

If we were to look upon the system as possessing potential energy U, where U is what f_2 becomes when the $\dot{\psi}$'s have been eliminated, and kinetic energy f_1, the equation would be

$$\frac{d}{dt}\frac{\partial f_1}{\partial \dot{\varphi}} - \frac{\partial f_1}{\partial \varphi} + \frac{\partial U}{\partial \varphi} = \Phi \tag{6}$$

These equations differ only in the fact that $\dfrac{\partial U}{\partial \varphi}$ occurs in one where $-\dfrac{\partial f_2}{\partial \varphi}$ appears in the other. But these two expressions are in reality equal. For since f_2 is homogeneous and quadratic in the $\dot{\psi}$'s,

$$2f_2 = c_1\dot{\psi}_1 + c_2\dot{\psi}_2 + \cdots = \Sigma c\dot{\psi}\cdots \tag{7}$$

c being the constant value of $\dfrac{\partial f_2}{\partial \dot{\psi}}$.

$$\therefore \frac{\partial f_2}{\partial \dot{\psi}} = \text{const.} = c$$

Since f_2 is a quadratic function of the $\dot{\psi}$'s each of the expressions $\dfrac{\partial f_2}{\partial \dot{\psi}}$ will be a linear function of the $\dot{\psi}$'s, whose coefficients are functions of the ψ's. The number of linear equations thus formed will be equal to the number of concealed coördinates. It will thus be possible to express each $\dot{\psi}$ in terms of the known coördinates and constants. Upon substituting these values in f_2 an expression is obtained which contains the known coördinates only.

Therefore
$$U = f_2 = \Sigma c\dot{\psi} - f_2 \tag{8}$$
$$\frac{dU}{d\varphi} = \Sigma c \frac{d\dot{\psi}}{d\varphi} - \frac{df_2}{d\varphi}, \tag{9}$$

the differentiation being complete. Since U contains the φ's only, and since further the generalized coördinates must be entirely independent,

$$\begin{aligned} \frac{dU}{d\varphi} = \frac{\partial U}{\partial \varphi} &= \Sigma c \frac{d\dot{\psi}}{d\varphi} - \frac{\partial f_2}{\partial \varphi} - \Sigma \frac{\partial f_2}{\partial \dot{\psi}} \frac{d\dot{\psi}}{d\varphi} \\ &= \Sigma c \frac{d\dot{\psi}}{d\varphi} - \frac{\partial f_2}{\partial \varphi} - \Sigma c \frac{d\dot{\psi}}{d\varphi} \\ \frac{\partial U}{\partial \varphi} &= -\frac{\partial f_2}{\partial \varphi}. \end{aligned} \tag{10}$$

It is thus seen that the equations of motion obtained on the assumption of a potential energy U are the same as those resulting from a complete and correct treatment of the problem. In any actual case there would really be nothing that would even suggest the existence of the concealed coördinates and the motions corresponding to them.

From this point of view the conception of potential energy is to be looked upon as resulting from the incompleteness of our knowledge concerning the mechanism of physical phenomena.

The above treatment of the subject, while perfectly general and conclusive, needs concrete illustration to make it fully appreciated by the average student. Numerous illustrations of concealed motion have been given, some of the common ones being the gyroscope and the Foucault pendulum. Both of these illustrations are excellent ones except for the fact that the analytical discussion of itself presents considerable difficulty. A problem or experiment which is to serve as an illustration of a general principle should, it appears to me, be in itself as simple as possible, so that the attention may be devoted to the principle which it is intended to illustrate, rather than to the difficulties of the problem itself. The case discussed below, which has been devised merely to illustrate the general principle already considered, is certainly so simple a mechanical problem that it should offer no difficulties. As an illustration of the principle it appears to me to be as satisfactory as any of the other cases usually employed.

At each end of a rod of length $2r$ is mounted a mass m, whose dimensions are so small as compared to the length r that the mass may be looked upon as concentrated at a point. The rod is pivoted at the center, as shown in the figure, so as to be carried by the arm R in its rotation about the verticle shaft SS. Attached to the shaft SS, either directly or by means of a belt, is a heavy fly wheel. The problem will be still

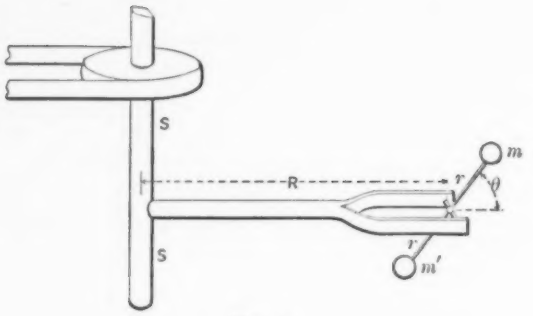


Fig. 1.

further simplified if inertia of this fly wheel is so great that the angular velocity, ω, of the shaft will not be appreciably modified by any change in the position of the rod rr and its attached weights.

On account of the manner in which the masses mm' are mounted, the potential energy of the above system will be constant, and therefore need not be considered. The variable part of the energy is wholly kinetic. If K represents the moment of inertia of the fly wheel and its attachments the expression for the kinetic energy will be

$$T = \tfrac{1}{2}K\omega^2 + mR^2\omega^2 + mr^2\dot{\theta}^2 + mr^2\omega^2\cos^2\theta \qquad (12)$$

It is evident that the rod rr will tend to set itself at right angles to the shaft SS; and if displaced from this position it will vibrate back and forth with a pendulum-like motion until brought to rest by friction. The exact character of this motion may be found by forming the Lagrangian equation, which, for the coördinate θ, becomes

$$2mr^2\ddot{\theta} + 2mr^2\omega^2\sin\theta\cos\theta = \Theta$$

where Θ is the generalized force acting from outside the system to increase the angle θ. If the system is free from outside influences the equation of motion for small vibrations about the position of equilibrium reduces to

$$\ddot{\theta} + \omega^2\theta = 0. \qquad (13)$$

The rod therefore vibrates with the period $2\pi\omega$.

Imagine now that an observer is carried around with the arm R in its rotation, the conditions being such that he is not only ignorant of the mechanism of the apparatus that carries him, but is actually unaware of the fact that he is moving. Such conditions are not ideal merely; they are readily obtainable, and in a variety of ways. The observer might, for example, be carried in a closed car which moves uniformly along a circular track. The rod rr, with its two weights m and m', should then be mounted so that its supporting shaft is parallel with the rails.

If such an observer, without knowledge of his own motion, were to study the behavior of the rod and weights, he would be almost certain to explain its motion wrongly. He would observe that the system had a definite position of equilibrium, and that when displaced from this position a force tended to make it return. By a systematic set of measurements he would find that the moment of this return force was proportional to the sine of twice the angle of displacement. It would certainly be most natural to assume that the system possessed potential energy when displaced, this energy being equal in amount to the work done during the displacement. Calling the potential energy U its value would therefore be

$$U = \int k \sin 2\theta d\theta = -\tfrac{1}{2}k\cos^2\theta + \text{const.} \qquad (14)$$

The kinetic energy assigned to the system would naturally be that of the visible motion only: *i. e.*,

$$T = mr^2\dot{\theta}^2 \qquad (15)$$

The equation of motion obtained by substitution in the general expression

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} + \frac{\partial U}{\partial \theta} = \theta$$

would become, in the case considered,

$$2mr^2\ddot{\theta} + k \sin\theta\cos\theta = \theta.$$

For small vibrations without any outside force this gives a period equal to

$$2\pi\sqrt{\frac{k}{2mr^2}}$$

But since the period as already computed is also equal to $2\pi\omega$ we obtain for the value of k in terms of the actual angular velocity ω

$$k = 2mr^2\omega^2$$

and therefore $$U = -mr^2\omega^2\cos^2\theta + \text{const.}$$

It thus appears that correct results are attained by assigning to the system, in addition to the kinetic energy $mr^2\dot{\theta}^2$,, an amount of potential energy equal to $-mr^2\omega^2\cos^2\theta + \text{const.}$; whereas, what the system really possesses is the additional *kinetic* energy

$$+mr^2\omega^2\cos^2\theta + \text{const.}$$

It would be entirely misleading, however, to leave the result in this form, for the above conclusion leads to an apparent violation of the principle of the conservation of energy. Upon the hypothesis that the energy is partly potential the expression for the whole energy of the sys-

tem would be

$$E = T + U = mr^2\dot{\theta}^2 - mr^2\omega^2\cos^2\theta + \text{const.}, \qquad (16)$$

while the energy is really

$$E = mr^2\dot{\theta}^2 + mr^2\omega^2\cos^2\theta + \text{const.} \qquad (17)$$

Now if we operate upon the system from without so as to actually pull the rod and weights out of their equilibrium position, real work is done, and the energy of the system must increase. The first expression above does increase by the amount of work done, but the second apparently diminishes. Cases more confusing than this, because of their greater complexity, often arise throughout all branches of physics, and are sometimes rather vaguely spoken of as illustrations of the fact that while "potential energy tends toward a minimum, kinetic energy tends toward a maximum." The following quotation[1] will serve as an illustration: "Two investigations in the second volume of Maxwell's Electricity and Magnetism afford a good illustration of the way in which this ambiguity[2] is cleared away by an increase in the precision of our ideas about the configuration of the system. In the early part of the volume by considering the mechanical forces between two circuits carrying electric currents, it is shown that two such circuits conveying currents i, j possess a quantity of potential energy $-Mij$ where M is a quantity depending on the shape and size of the two circuits and their relative position. Later on, however, when coördinates capable of fixing the electrical configuration of the system have been introduced it is shown that the system instead of possessing $-Mij$ units of potential energy really possesses $+Mij$ units of kinetic."

As it stands the above statement is certainly misleading. For let us imagine that the circuits are compelled to move apart against their mutual attraction. Work has to be done in order to accomplish this, and in consequence the energy of the system of currents *must* increase. As far as this fact is concerned it is quite immaterial whether we look upon the energy as potential or kinetic. It is manifestly impossible, however, for any movement of the system to cause an increase both in $+Mij$ and in $-Mij$. We are forced to conclude, therefore, either that one of the two expressions is obviously wrong, or else that the statement of the matter is incomplete.

The difficulty is, of course, readily explained. In the case of the system of electric currents for example, the potential energy is properly expressed by $-Mij$ only when the currents are constant[3]; for if i and j

[1] J. J. Thomson, Applications of Dynamics to Physics and Chemistry, p. 12.

[2] Namely the difficulty of distinguishing between the two kinds of energy.

[3] $-Mij$ is the proper expression for the mutual potential energy of two magnetic shells which have the same boundaries as the circuits in question, and strengths equal

are allowed to vary as the result of motion, the expression $-Mij$, although in reality a function of position only, is not such in form. To keep the currents constant requires the presence of some source of variable electromotive force, such as a battery or dynamo, which is able to supply energy to the circuits or to store energy received from them. It is the fact that this essential part of the system is left out of account that leads to the apparent violation of the energy principle.

A similar explanation applies to the mechanical case.[1] The energy stored by the fly wheel, and given up by it, can not be neglected, even when the apparatus is so massive that the variation in the angular velocity, ω, is insignificant. In other words the term $\frac{1}{2}K\omega^2$ in the energy expression cannot be treated as a constant. If the variations in ω are to be small, this requires that K shall be large; the product $\frac{1}{2}K\omega^2$ will therefore be subject to changes which are clearly comparable in magnitude to the changes in the other terms in the expression for T. ω may be treated as a constant in the last two terms of (12) but cannot be so treated in the first.

In a more rigorous treatment of the mechanical problem the general method outlined on p. 107 may be applied with advantage. ω is in this case the velocity of the concealed coördinate. Since no forces act from outside to change ω we have:

$$\frac{\partial T}{\partial \omega} = \text{const.} = (K + 2mR^2 + 2mr^2\cos^2\theta)\omega = c. \qquad (18)$$

Solving for ω and substituting in (12)

$$T = mr^2\dot{\theta}^2 + \frac{c^2}{2(K + 2mR^2 + 2mr^2\cos^2\theta)}. \qquad (19)$$

The original expression for T was

$$T = mr^2\dot{\theta}^2 + \tfrac{1}{2}(K + 2mR^2 + 2mr^2\cos^2\dot{\theta})\omega^2 \qquad (20)$$

The two expressions are of course equal. But in (19) the concealed coördinate has been eliminated. Equation (20) therefore corresponds to (3), while in (19) the term f_2 has been replaced by U. The fact that $\frac{\partial f_2}{\partial \theta} = -\frac{\partial U}{\partial \theta}$ is readily verified.

to the two currents. The mechanical *force* between two such shells is the same as the force between the two electric circuits; but to assume that the *energy* of the system of shells is the same as that of the currents is to take a step whose soundness is at least open to question. If, however, the currents are constant, no matter how this constancy is attained, $-Mij$ properly represents the mutual energy so far as this depends upon the ordinary mechanical forces between the circuits.

[1] If the angular velocity, ω, of the shaft SS were kept constant by some outside help, the analogy between the mechanical and electrical cases would be quite striking.

It will be observed that when K is large,[1] in which case ω is practically constant—

$$U = \frac{c^2}{2(K + 2mR^2 + 2mr^2\cos^2\theta)},$$

$$= \frac{c^2}{K} - \frac{c^2}{2K^2}2mr^2\cos^2\theta + \cdots,$$

$$= \frac{c^2}{K^2}mr^2\cos^2\theta + \text{const.},$$

since the terms involving higher powers of $\frac{1}{K}$ become insignificant. When K is large the constant factor $\frac{c^2}{K^2}$ is also practically equal to ω^2 and the expression for U takes the form $-mr^2\omega^2\cos^2\theta + \text{const.}$, which is the same as that previously found. In this expression ω^2 is to be treated as a constant.

It is thus seen that although the correct equations of motion are obtained by assigning to the system the potential energy $-mr^2\omega^2\cos^2\theta$, we are not justified in saying that this energy *takes the place* of the kinetic energy $+mr^2\omega^2\cos^2\theta$. The term $-mr^2\omega^2\cos^2\theta + \text{const.}$ in reality takes the place of the two terms $\frac{1}{2}K\omega^2 + mr^2\omega^2\cos^2\theta$ in the energy expression, and the value of the total energy remains unaltered, as it should, no matter which assumption is made regarding the distribution of energy between the two forms, kinetic and potential.

Very little novelty can be claimed for the preceding discussion. But the very simplicity of the mechanical illustration makes it appear a desirable one to use in explaining some important and rather abstract dynamical principles. It is needless to say that the analogy with certain well-known laws in the dynamical theory of electricity can readily be carried much further than has been done here.

[1] In this case $2mR^2$ may be neglected in comparison with K.

The Resistance of the Human Body to Direct and Alternating Currents.

By James E. Boyd.

THE object of the experiments described below was to obtain the resistance of the living human body to relatively large currents of electricity, both alternating and direct, and to determine whether this resistance is constant with currents of varying intensity.

For the first part of the work a Duncan dynamometer was used, which was provided with special fixed coils having a great number of turns, making it sufficiently sensitive to measure the currents used. The calibration of the instruments showed that the deflections were practically proportional to the square of the current strength, and that its constant was .00084 amperes.

The method employed was to take a reading with a dynamometer in series with a non-inductive resistance of 1550 ohms, and then throw the instrument in series with the resistance to be measured and take a second reading. The results given below were from hand to hand. Connection was made to the circuit through a couple jars of sal-ammoniac solution. The four fingers of each hand were thrust down into a jar of the solution so that they were just covered by the liquid. The hand above the surface was kept dry. The connection between line and solution was made by means of large battery carbons. Measurements taken with both carbons in one jar showed that the resistance of the carbons and liquid could be neglected. It was also found that, with the currents used, the polarization at the carbons was inappreciable.

The table below gives results with sal-ammoniac of density 1.035 and also with a concentrated solution. These were taken with direct currents and with alternating currents of 62 periods per second.

The first column gives the reading through the resistance of 1550 ohms and the dynamometer, whose resistance is 856 ohms. The second is the reading through the body from hand to hand. The third gives the square root of the ratio of the first to the second. The figures of the fourth column were obtained by multiplying the corresponding numbers of the third by 2406 and subtracting 856 from the product. The fifth column gives the amperes through the body, and the sixth, the potential difference of the terminals in the carbon plates.

	I.	II.	III.	IV.			
	Reading through 1550 ohms.	Reading through body.	$\sqrt{\frac{I}{II}}$	Resistance.	Current.	E.M.F.	
Alternating current.	10.0	10.0	1.00	1550	.0026	4.0	Fingers in NH_4Cl 1.035.
	15.2	15.0	1.01	1574	.0032	5.0	
	19.0	20.0	.98	1502	.0039	5.8	
	17.0	16.0	1.03	1622	.0034	5.5	
	27.0	27.0	1.00	1550	.0044	6.8	
	27.0	27.0	1.00	1550	.0044	6.8	
			Mean.	1558			
Direct current.	12.0	8.0	1.22	2079	.0024	5.1	
	21.0	16.0	1.14	1887	.0033	6.2	
	30.0	29.0	1.02	1598	.0045	7.2	
	41.0	38.0	1.04	1646	.0052	8.6	
	12.0	10.5	1.07	1718	.0027	4.6	
			Mean.	1786			
Alternating current.	3.2	3.3	.98	1502	.0015	2.2	Concentrated NH_4Cl.
	5.5	5.8	.97	1478	.0020	2.9	
	8.8	9.1	.98	1503	.0025	3.7	
	14.5	15.0	.98	1502	.0032	4.8	
			Mean.	1497			
Direct current.	9.9	8.2	1.09	1766	.0024	4.2	
	17.0	14.5	1.08	1742	.0032	4.6	
	24.0	21.5	1.06	1644	.0039	6.4	
	19.1	17.2	1.05	1670	.0035	5.8	
	14.3	13.0	1.05	1670	.0030	5.0	
			Mean.	1698			

The dynamometer has considerable self-induction, so that the current lags when it is used in series with the non-inductive resistance. On the other hand, if there be any electrolytic action in the body it acts as a capacity introducing a lead. If θ be the angle of lag when the non-induction resistance is used, and φ, the angle of lead or lag with the body in the circuit, while R is the resistance of the body and ρ the ratio of column III, we have

$$R + 856 = 2406\,\rho\,\frac{\cos\varphi}{\cos\theta}.$$

The calculations of the table were made on the assumption that $\frac{\cos\varphi}{\cos\theta}$ is equal to unity. To see if this assumption is warranted, we measured the self-induction of the dynamometer by the bridge method and computed the value of cosine θ which we found to be .999.

To get at the value of the angle φ we connected the primary of a transformer containing no iron to one coil of a dynamometer, and the secondary to the other. The secondary E. M. F. in this case is 90° behind the primary current. When the dynamometer coil alone was in this secondary circuit there was a deflection to the right indicating a lag due to the self-induction of the coil itself. When the resistance of the body was put in series with it the deflection was to the left indicating a lead. As was to be expected in the case of an electrolyte, this angle of advance was found to diminish as the E. M. F. increased. Computed on the assumption that the resistance of the body is 1500 ohms, the results were:

Angle of Advance.	E. M. F.
10°45′	2.16
7°45′	2.70
6°00′	3.53

When the dynamometer is in series with the body the angle of advance of current ahead of total E. M. F., as computed from the largest one of these figures above, is 4°30′. This angle has the cosine .997. So we find that the correction $\frac{\cos \varphi}{\cos \theta}$ may be neglected.

As far as these results go, they show that the resistance measured with direct currents is larger than when measured with alternating currents; and that with the alternating currents it is independent of the voltage.

The ordinary fall of potential method with direct current, which has been shown by Ayrton[1] to apply to electrolytes, was used for the remainder of the work. The current was measured by means of a Weston standard voltmeter and the drop of potential with a condenser and galvanometer. Platinum foil strips fastened to the points between which the resistance was to be measured, served as the terminals of the condenser. With the terminals in place, a reading of the condenser discharge was made with no current through the body. If this was found to be large, the strips of foil were taken off and heated to redness and again put back. If small, the readings were next taken with the current flowing; then another reading without a current. The mean of the readings with no current subtracted from the reading with a current flowing gave the fall of potential required.

The table below is an example of the method. The current came in through the fingers as in the previous experiment, and the fall of potential was measured from wrist to wrist.

[1] See Ayrton and Perry, Proceedings of Physical Society, 1883, p. 303.

Current.	Deflection.	Deflection due to fall of Potential.	Volts.	Resistance.
.0	−2			
.00226	96	97.5	1.404	628
.00221	95			
.0	−2			
.00226	95			
.00226	101	100.5	1.447	637
.0	−3			

Three other sets of readings with 0008 ampere gave 629, 646, and 604 as the resistance. In the same way from the left jar of liquid to the left wrist, the figures obtained were 370 and 353. From the right wrist to the liquid, the results were 337 and 349. Adding the mean values we get 1334 ohms as the resistance from one jar to the other. This is less than the results obtained in the other way, and is probably due to the fact that the hands were not thoroughly dry above the surface of the liquid.

In taking readings from the solution to the body a platinum wire projecting from a glass tube was used instead of the platinum foil. The results were seldom as satisfactory as those obtained from one part of the body to the other. From the right wrist to the elbow the resistance was found to be 200 ohms. From the wrist to the end of the shoulder blade, it is 267. From shoulder to instep it is 291. These last figures are independent of the skin resistance which is inversely proportional to the area, and varies with the degree of moisture and the nature of the electrode.

OHIO STATE UNIVERSITY, July 8, 1898.

The Orbital Motion of a Steel Ball Around a Magnetic Pole.

By R. W. Wood.

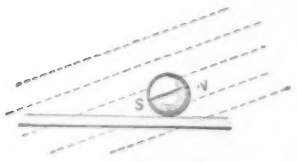
In Volume IV., No. 5 of the Physical Review I described an experiment for illustrating orbital motion by projecting a steel bicycle ball over a glass plate, perforated by the pole-piece of a large electro-magnet.

Several criticisms appeared in *Nature* and elsewhere, claiming that, under the conditions, the attractive force would vary not as the inverse square, but as the inverse fifth power of the distance. The authors of these notes appear to have overlooked the fact that the ball does not move in along a line of force, being urged towards the center by the horizontal component only, the vertical producing pressure on the plate. If the plate is over the magnetic pole this component becomes zero at the center instead of ∞, which would of course be the value in an hypothetical case only.

With a view of determining whether some position of the plate could be found, giving to the horizontal component a law approximating the law of gravitation within a certain area, I suggested to one of our students an experimental study of the attractive force between the magnet and the ball moving in on different planes. These experiments Professor Austin has already alluded to in a letter recently published in *Nature*, but it seems worth while to give the results a little more in detail, together with their bearing on the experiment.

The first experiments were made with the magnet in a horizontal position, and the ball suspended against a vertical glass plate, from a pan of a balance placed high above the magnet. This method proved unsatisfactory owing to the friction caused by the pressure of the ball against the plate.

Very good results were obtained, however, by fastening the ball to a strip of tempered brass, and measuring the deflections by means of a mirror and scale. A large number of series of observations were made of which two are given below.



The pole of the magnet was at some little distance within the end of the bar. The first series shows the variation in the attractive force when the ball moved in on a plane 3 cms. above the pole. x is the distance from

the axis of the magnet, y the deflection which is proportional to the attractive force.

Distance of Plane from Pole, 3 cms.		Plane passing through Pole.	
x	y	x	y
4	2.10		
5	1.37	4	18.98
6	.91	6	5.79
7	.69	7	3.42
8	.51	8	2.14
9	.40	9	1.32
10	.31	10	.93
12	.20	12	.45
14	.12		

Calculating the attractive force for each of these two conditions from the formula $y = \frac{c}{x^a}$ when c and a are unknown constants we find that for the first series a has a value of about 2.1 indicating an attractive force earing very nearly as the inverse square. For the second series, where the plane in which the ball moves passes through the pole, the value of a is about 3, the force being as the inverse cube. In both cases the *law* of attraction changes from point to point, which is what one would expect, but within the region in which the orbits are uusally contained, it is approximately that of the inverse square or cube depending on the position of the plane.

These results indicate that planetary motion would be better represented on a plate a little above the pole piece, and this I found to be the case. With the plate at a distance of 3 cms. above the pole I have succeeded in making the ball make two revolutions in an elliptic orbit. I have abandoned the smoked plate as the friction is greater, and the numerous failures are very exasperating when each records itself, and practically spoils the appearance of the plate. The motion of the ball can be seen from a considerable distance and the experiment is on the whole more striking. I find that a grooved inclined plane is the best device for starting the ball, as its direction can be easily adjusted and the velocity varied at will. It is a good plan to draw the temper of the steel by heating the ball red hot over a Bunsen burner and cooling it slowly. On projecting the ball with moderate velocity along a line passing some 20 cms. from the pole I find that it will often complete five or six complete revolutions in a very large orbit, moving slower and slower and eventually coming to rest without "falling into the sun." This is a very striking experiment, as it is of six or eight seconds' duration, but it is best to stop the ball before it comes to rest, if one wishes to palm it off as planetary motion! It is, of course, not a case of true orbital motion,

though none the less interesting. It is due to the permanent magnetization of the ball, the directive force of the field preventing it from rolling either towards or away from the pole. The ball sets itself with its magnetic axis along the lines of force, which make an angle with the plate, as shown in the figure, and if it rolls a little either to the left or right the directive force will roll it back. This can be shown by pushing it a trifle either towards or away from the center after it has come to rest. In both cases it rolls back at once to its original position. It has been practically rolling in a magnetic groove, if one may use the term, the *attractive* force playing no part. The temper of the steel should not be drawn for this experimens.

PHYSICAL LABORATORY OF THE UNIVERSITY OF WISCONSIN, MADISON,
June, 1898.

THE ELECTROMAGNETIC EFFECT OF ELECTROLYTIC CURRENTS.

BY SAMUEL SHELDON AND G. M. DOWNING.

OUR knowledge of the effects of magnetism upon the conditions of electrolysis has been amplified in recent years by Rowland, Nichols, Franklin, Squier, and others. Little attention, however, has been given to the investigation of the magnetic field as produced by an electrolytic current. The object of the authors primarily was to determine whether a current passing through an electrolyte produced magnetic influences of the same magnitude as a current of equal strength passing through a metallic conductor under the same conditions. According to modern theory, conduction in an electrolyte takes place by convection, which cannot be true of conduction in a metallic conductor. Since this difference exists, the authors thought it possible that the magnetic effects might also differ.

The arrangement of apparatus was essentially the same as that used in determining the magnetic qualities of iron by the method of reversals. Upon a soft iron annular test ring, which had been thoroughly insulated with tape, was wound a coil of 55 turns of No. 16 wire. This coil served as a secondary test coil and was connected to a sensitive ballistic galvanometer. Upon this was wound a coil of rubber tubing, 386 centimeters long and of about ¾ square centimeter internal cross section. This tube was filled before winding with a 30% solution of H_2SO_4. This concentration was taken as corresponding closely to the concentration for maximum conductivity. Considerable difficulty was experienced in maintaining the continuity of the electrolytic circuit. The introduction of air bubbles had to be carefully avoided as well as the formation of kinks in the tube. The end was attained, however, by filling the tube carefully, before winding, and closing its ends tightly by stoppers. After the winding was completed, these stoppers were removed and copper wire electrodes were inserted, which allowed the egress of the gases formed by electrolysis. This formed a primary coil of 15 turns, which was connected in series with a milli-ammeter to a commutator arranged so that the current could be reversed through the electrolyte. Since the resistance of this primary was over 700 ohms, it was convenient to use a voltage of 230, which was supplied from the street mains. When the secondary was connected to the ballistic galvanometer, the apparatus was ready for use. The average throw for four reversals of current at two

values of current strength was taken; an average reading of the ammeter was also determined.

Next, the electrolytic primary was replaced by 10 turns of No. 16 wire and a current, sufficient in strength to give the same number of ampere turns as previously, was used. The average of four throws of the galvanometer was taken as before. The following table gives the results: —

Ampere-turns.	Average throw with electrolytic primary.	Average throw with metallic primary.
5.1	9.0 mm.	9.0 mm.
5.4	9.7 mm.	9.4 mm.

While our experiments have been but hastily performed and with no pretensions to great accuracy, still, even allowing for all errors, our results seem to indicate an inconsistency between the accepted ideas of the two kinds of conduction—electrolytic and metallic as based upon what was formerly called the two-fluid theory. On the other hand, they seem to be in accord with a single-fluid theory, where all electrical phenomena are supposed to result from a single positive etheric disturbance. For if, according to the two-fluid theory, electrolytic conduction in H_2SO_4 consists in two oppositely directed progressions of oppositely charged ions having different velocities of migration, and if magnetic influences are due[1] to traveling electrostatic charges, the magnitude of these influences depending upon the velocity of movements, the magnitude and signs of the charges, and, furthermore, if metallic conduction is considered as consisting in two oppositely directed processions of oppositely signed charges with equal velocities, the magnetic intensities resulting from a certain number of ampere-turns of electrolytic conductor should be different from those due to the same number of ampere-turns of metallic conductor. On the other hand, if, according to the theory of a single positive etheric stress, metallic conduction consists of a single unidirectional progression of charges and (for the sake of consistency) but one set of ions be considered as carrying a magnetically active charge, electrolytic ampere-turns should produce the same effect magnetically as the same number of metallic ampere-turns.

The utilization of these terms—one- and two-fluid theories—while savoring of ancient history, are used here because the ether theory has not supplied us with terms which can fully replace them as vehicles of thought. Furthermore, the ether theory has not yet given to the imagination a satisfactory picture of the process of either metallic or electrolytic conduction.

[1] Phil. Mag., 1876, Vol. II., p. 233.

NEW BOOKS.

Luce e Raggi Roentgen. By ORESTE MURANI; with a preface by R. FERRINI. Large 8vo, pp. x, 392. Milan, Ulrico Hoepli, 1898.

The great interest taken in X-ray phenomena, both from the scientific and popular standpoint, has led to the publication in recent years of a number of books devoted to this subject. The general phenomena of the electrical discharge in rarefied gases have usually been described by way of introduction, and in many cases the subject of light has also been briefly presented. These books have naturally been popular in character.

Murani's work on "Light and Roentgen Rays" possesses the same general characteristics. The first two chapters are devoted respectively to "Light" and the "Discharge of Electricity in Rarefied Gases," and the later chapters lay more stress upon the popular side of the subject than would be expected in a purely scientific treatise. But the book nevertheless has distinct characteristics of its own, and although strictly a semi-popular presentation of the subject, will also be found of value to those whose interest lies on the scientific side. It is clearly written and shows few of the indications of hasty preparation that are often found in some of the earlier books on this subject. At the close of each chapter is a list of references to original sources.

Under the comprehensive title "Light," the first chapter not only deals with the visible rays, and to some extent also with the infra-red and ultra-violet portions of the spectrum, but also contains a short account of the essential phenomena of electrical waves and oscillations. This chapter makes no attempt to take the place of a treatise on optics. The reader is supposed to have already acquired a reasonable familiarity with the general subject. The author's object seems to be to call to mind afresh those phenomena that will be of especial interest, either by similarity or contrast, in the discussion of X-rays that is to follow. The attention is directed especially to such subjects as selective absorption, interference and diffraction, polarization, anomalous dispersion, and fluorescence, and the modern views regarding the nature and causes of these phenomena are clearly stated.

The second chapter, which is devoted to the electrical discharge in rarefied gases, is in many respects the most satisfactory short presentation of this subject with which I am familiar. The subject is not an easy one

to present; for the great mass of experimental results, which in general are only loosely connected by any satisfactory underlying theory, renders the choice of material for a brief discussion especially difficult. In a semi-popular presentation, like that contained in the work of Murani, the logical arrangement of the material chosen presents perhaps even greater difficulties. Yet in the brief space of about sixty pages a remarkably complete account is given of the most important experimental work up to the date of publication, and the direction in which such work is tending is clearly indicated.

To those of us who have gained our knowledge of vacuum-tube phenomena chiefly from English books, this chapter presents many novel features. In reading it we obtain a view of the subject from the Italian standpoint, a view which is certainly broader than that offered by most books that have appeared in English. This is perhaps explained by the fact that until recently the experimental work carried on in Italy was small compared to that of the active scientific nations. It was necessary for the very scientific existence of the Italian physicist to keep in touch with the work of foreign scientists; and the habit formed still remains even when Italy has taken a prominent place among the nations which are adding to the world's stock of scientific knowledge.

While giving what seems to us unusual prominence to the work of continental physicists, Murani does not ignore the investigations of English workers along this line. In addition to a rather complete account of the work of Crookes, mention is made of the experiments of J. J. Thomson upon electrolytic conduction in gases, and of his earlier experiments upon the velocity of kathode rays. Unfortunately the more recent work upon this subject came out too late to be included.

The unusual features of the chapter begin with the brief description of some of Hittorf's earlier work (p. 104). In the small space available it is of course impossible to do justice to the extensive and important investigations of this great pioneer in the systematic study of vacuum-tube phenomena. But the reader is at least made aware of his existence; and a reference at the close of the chapter makes it possible to consult, if desired, the first, and perhaps the most interesting, of his series of articles. One cannot help thinking that familiarity with this paper, published in 1869, might have spared the makers of X-ray tubes many expensive experiments.

The treatment of kathode-ray phenomena is in general that of Crookes; but the more recent experiments are frequently referred to. Mention is made, for example, of the experiments of Wiedemann and Ebert on the mutual electrostatic action of two bundles of kathode rays. They found that the repulsion of parallel kathode rays, which is illustrated in one of the well-known experiments of Crookes, is repulsion in appearance only,

and really results from causes quite different from those usually assigned. A more serious shock awaits the orthodox reader on p. 120, where a figure is shown to illustrate the shadows cast in a bundle of kathode rays from a concave kathode. The object being placed between the kathode surface and its center of curvature, the shadow is found to be *erect* instead of inverted.[1] In other words kathode rays do *not* always proceed along the normal to the kathode surface; and once started their path is *not* always absolutely straight, even when they are free from outside magnetic or electrostatic influences.

Among the other unusual features in the treatment of kathode rays may be mentioned the brief description of the experiments of Goldstein upon the reflection of these rays. The work of Hertz upon the magnetic effect of kathode rays is also referred to. A drawing is reproduced from his paper on this subject which shows the lines of flow in a vacuum tube as determined by magnetic measurements. It is seen that the current passes between the two terminals with practically the same distribution as one would expect if the tube were filled with some liquid electrolyte; the lines of flow are quite uninfluenced by the kathode rays. The magnetic effect of the latter is therefore insignificant, and only a small fraction of the total current can be carried by them. At the end of the chapter is a description of Lenard's experiments on kathode rays that have passed outside the tubes in which they are developed. Lenard's original drawing is reproduced to illustrate the now well-known "magnetic spectrum." The chapter closes with the experiments of Perrin showing that the kathode rays carry a negative charge.

The consideration of the X-rays themselves is begun in Chapter III. and continues through the remaining two hundred and fifty pages. A general account of Roentgen's discovery and earlier work naturally comes first. In speaking of the original discovery Murani says that "it was while repeating the experiments of Lenard, of which I have spoken in the preceding chapter, that Professor Roentgen was accidentally led to the discovery of the new radiation." This very natural explanation of how the discovery came to be made has doubtless occurred to almost all who have given the matter thought. But I have not before seen any confirmation of the view. The statement of Professor Murani, who doubtless speaks with authority, is therefore of especial interest.

At the close of Chapter III., which is devoted almost wholly to the work described in Roentgen's first paper, the author refers to the essential differences between the X-rays and the kathode rays of Lenard, and points out that the resemblance between the two is largely superficial.

[1] This is true only at high exhaustions. When the pressure is so high that kathode rays are just beginning to be formed the shadow is inverted. See Goldstein, Wied. Ann., 15, p. 254, 1882.

In this connection it should be remembered that Roentgen in his third communication[1] is inclined to give more weight to the resemblances between the two classes of rays than he did in his first papers. Undoubtedly the present tendency is toward a theory which makes a sharp distinction between kathode rays and Roentgen rays. But the time has hardly arrived yet when their resemblances can be looked upon as devoid of significance.

In Chapter IV. are described some of the attempts to obtain regular reflection, refraction, diffraction, and polarization. A few pages are also devoted to the experiments made to determine the place at which the X-rays are developed. During the six months immediately following Roentgen's discovery probably no one phase of the subject received so much attention as this question regarding the source of the rays. Some writers took it for granted that they came from the kathode, and cited confirmatory experiments; others made them come from various parts of the glass; still others traced the rays to the anode, or to some point in empty space between the two terminals; in fact, in the struggle for the distinction of being the source of this new radiation, practically every point and particle of the tube had its group of ardent champions. And then finally some one happened to read Roentgen's original paper, where the answer to the question was clearly and definitely stated! It is such cases as this that illustrate the wonderful excitement caused by Roentgen's discovery. Everyone wished to *do* something with the new rays; to read what others had done was in comparison tedious and uninteresting. Murani has wisely made no reference to the greater part of the experiments on this subject, and in addition to Roentgen's experiments cites only those of Róiti.

In Chapter IV. considerable space is devoted to a description of the various forms of tubes for X-ray work. This chapter also contains a discussion of the connection between the fluorescence developed by kathode rays and the simultaneous production of X-rays. This naturally leads up to the discovery of Becquerel rays. But the extended discussion of the latter subject is postponed until a later chapter.

After a brief account of the photographic action of the X-rays and of the experiments dealing with the transparency of various substances for the new radiation, we come in Chapter V. to the subject of the electrical action of the rays. The enormous stimulus given to experiment by the announcement of Roentgen's discovery is nowhere better illustrated than in this branch of the subject. For it will be remembered that, within two months after the appearance of Roentgen's first paper, the fact that electrified bodies are discharged by the action of X-rays was in-

[1] Wied. Ann. 64, p. 18, 1898. Murani does not refer to this article; it doubtless appeared after his book had gone to the press.

dependently discovered by at least five different physicists, representing five different nations and languages. Murani describes first the original experiments of Benoist and Hurmuzescu, J. J. Thomson, Righi and Roentgen. He then takes up the various phases of the subject in detail; for example, the effect upon solid and liquid dielectrics; the conducting power imparted to gases, and its dependence on pressure; electrical methods of measuring the intensity of X-rays. As far as the earlier experiments are concerned the account is very complete. But since some of the most important work in this field has appeared within the last six months, this chapter fails to give the reader the most recent views on the subject.

In discussing the nature of X-rays (Chapter VIII.) the author is conservative, and gives a brief account of most of the best known theories without showing preference. In fact throughout the book prominence is given to the descriptive rather than to the theoretical side of the subject. With regard to the nature of kathode rays, whose bearing upon X-ray theories is of such obvious importance, it is interesting to note that the author evidently agrees with the modified Crookes theory, which is now so universally accepted. This is the more surprising when we remember that the book was written at a time when the conclusive experiments of Kaufmann, J. J. Thomson, and Lenard had not yet been published.

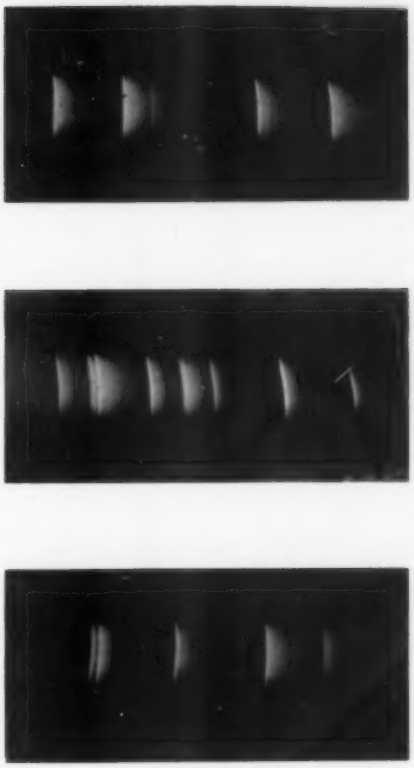
A number of radiographs are reproduced throughout the book, and at the end are several full-page plates illustrating the usual subjects. The paper is unfortunately not such as to show these half-tones at their best. Diagrams are extensively used in all parts of the volume.

Enough has been said, I think, to show that we have in Professor Murani's book a valuable contribution to the literature of the Roentgen rays. The work is one which would well repay translation. But the rapidity with which the subject is progressing will probably prevent such an undertaking. Rapid scientific progress is certainly not to be regretted; but we may at least be permitted to sympathize with the writers in this field, whose books must often run the risk of becoming antiquated even before they leave the press.

ERNEST MERRITT.

The following plates should accompany the article by W. P. Boynton, entitled "A Quantitative Study of the High-Frequency Induction Coil." [PHYSICAL REVIEW, VOL. VII., No. 1.]

(To face page 50.)



Boynton: High-Frequency Induction Coil.

(Photographs of the Oscillatory Spark.)

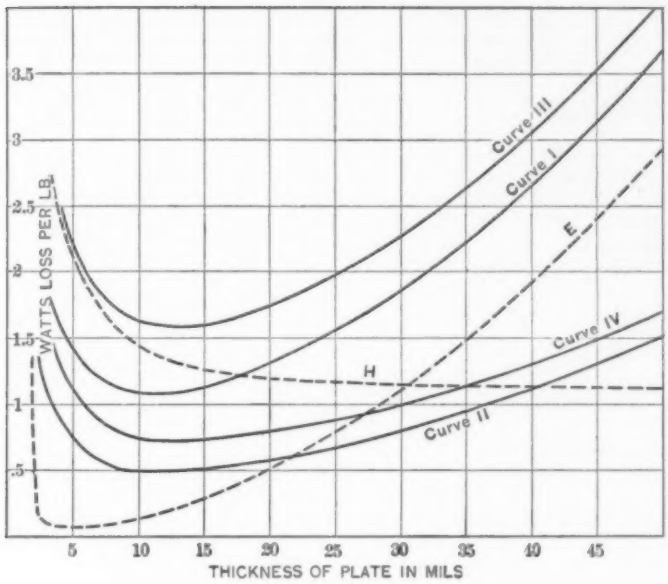


Fig. 1.

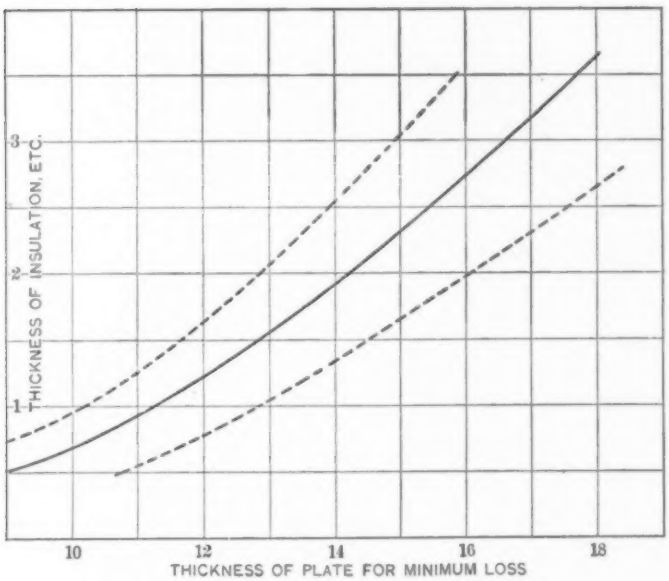
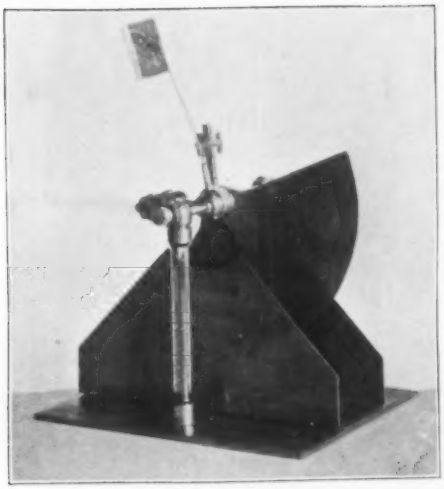


Fig. 2.

LIBRARY OF THE MICH.

(To Face page 53.)



Boynton: High-Frequency Induction Coil.

(Electrometer.)

U. MICH.